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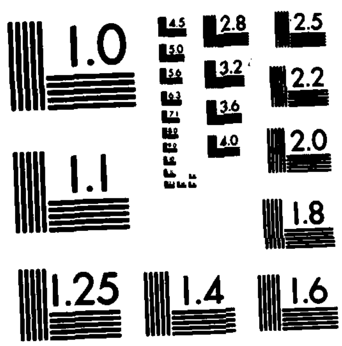

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A MODIFIED KOLMOGOROV-SMIRNOV, ANDERSON-DARLING, AND CRAMER-VON MISES TEST FOR THE GAMMA DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE PARAMETERS

THESIS

AFIT/GOR/MA/82D-4

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The Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises statistics are used to develop a new test of fit for the three-parameter gamma distribution with unknown location and scale parameters. The goodness-of-fit tests are valid for sample sizes, n = 5,10,...30, and shape parameter, K = .5,1.0,...4.0.

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A power investigation of the tests is performed against ten alternative distributions. The most powerful test against the alternative distributions is the Cramer-von Mises, followed closely by the Anderson-Darling; the Kolmogorov-Smirnov is the least powerful.

A functional relationship between the critical values and shape parameter is investigated for each test. The critical values can be expressed as a function of the inverse of the square of the shape parameter.

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fulfillment of the
Requirements for the Degree of
Master of Science

by

Philip J. Viviano
Capt USAF

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Preface

Goodness-of-fit tests are developed for the gamma distribution when the scale and location parameters are unspecified and must be estimated from the sample data. The Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling statistics are used to develop the tables. A comprehensive power study is conducted to compare the Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling goodness-of-fit tests. An analysis is performed to determine the functional relationship between the critical values and the shape parameter.

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Finally, I would like to express my appreciation to my classmates for their help and encouragement during the preparation of my thesis.

Philip J. Viviano

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Abstract

The Anderson-Darling, Cramer-von Mises, and the Kolmogorov-Smirnov statistics are used to develop a new test of fit for the three-parameter gamma distribution with unknown shape and location parameters. The critical values generated were obtained by a Monte Carlo procedure. For each value of n (sample size), 5000 sample sets were drawn from a gamma population whose shape is specified. The location and scale parameters are estimated from the data, and the three statistics are calculated based on the estimated distribution. The simulation was preformed for sample sizes n = 5, 10, 15...30 and shape parameters, K = .5, 1.0, 1.5...4.0.

Using gamma distributions for shape equal to 1.5 and 4.0, the power of each test is investigated against ten alternative distributions for sample sizes n = 5, 15, and 30. In general both the Anderson-Darling and the Cramer-von Mises tests are more powerful than the Kolmogorov-Smirnov test. Except for the case where the alternative distribution is lognormal, the Cramer-von Mises test is the most powerful test.

The functional relationship between the critical values of the Anderson-Darling, Cramer-von Mises, and Kolmogorov-Smirnov is also examined. A critical value for a shape parameter between 1.5 and 4.0 which is not included in the tables can then be easily derived from this functional relationship.

A MODIFIED KOLMOGOROV-SMIRNOV, CRAMER-VON MISES AND ANDERSON-DARLING TEST FOR THE GAMMA DISTRIBUTION WITH UNKNOWN LOCATION AND SCALE

I. Introduction

Currently the U.S. Air Force is placing more and more emphasis on system availability, maintainability, and reliability, both in research and development and in day to day operations. Of particular importance to the Air Force is the ability to predict time-to-failure of equipment. Studies in probability and statistics have increased understanding of some key probability distributions used in predicting time-to-failure. Among the most commonly used continuous distributions in this area are the beta, gamma, exponential, Weibull, and lognormal distributions.

Often in these studies, analysts are confronted with the problem of testing agreement between probability theory and actual observations. In other words, given n observations of some variable, say time-to-failure, the problem is to find out if it can be regarded as a random variable having a given probability distribution. The general approach to the solution of this problem is known as the goodness-of-fit test. In more precise terms let $x_1, x_2 \ldots x_n$ be a random sample. Then a statement of the goodness-of-fit test is:

$$H_0: \quad F(x) = F_H(x)$$

$$H_A: \quad F(x) \neq F_H(x)$$

where $F(x)$ is the actual distribution function of x and $F_H(x)$ is the hypothesized distribution function.

Background

Two commonly used goodness-of-fit tests are the Chi-square test and the Kolmogorov-Smirnov test. The Chi-square test compares observed frequencies with expected frequencies of the hypothesized distribution. It is restricted to large samples--approximately 25 or greater (2:73). The Kolmogorov-Smirnov (K-S) test compares cumulative frequencies between the actual sample using a step function, against corresponding values using the hypothesized cumulative distribution function. The K-S test can be used for large or small samples; however, it is restricted to distributions which are fully specified. H. W. Lilliefors developed a goodness-of-fit test for the normal (19), and exponential distribution (20), which can be used for small samples where the parameters must be estimated from the sample data. When parameters are estimated from sample data, the test is said to be a modified test.

Following Lilliefors' technique, several other modified tests have been documented. R. Cortes developed a modified Kolmogorov-Smirnov test for the three-parameter Weibull and gamma distributions (5). J. Bush expanded the goodness-of-fit test for the Weibull to include the modified Cramer-von Mises (W^2) and Anderson-Darling (A^2) tests (30). The modified Kolmogorov- Smirnov, Cramer-von Mises, and Anderson-Darling tests have also been done for the uniform, normal, Laplace, exponential and Cauchy distributions (11). In 1973 Mann, Scheuer, and Fertig designed two new test statistics

called the L and S statistics. The L and S test statistics were used to develop a goodness-of-fit test for the two parameter Weibull with unknown parameters (22).

In 1981 Koutrouvelis and Kellermeier introduced a goodness-of-fit test based on the empirical chacteristic function when the parameters must be estimated (17). This test statistic could be used as an alternative to the EDF statistic if the characteristic function is more easily determined than the distribution function.

<u>Empirical Distribution Function Statistics</u>

A general class of statistics used for the goodness-of-fit tests is called empirical distribution function (EDF) statistics. Historically EDF statistics have been used in cases where the parameters are either known or unknown. In most instances EDF statistics are easily calculated and are competitive in terms of power. This class of statistics is based on a comparison between the cumulative distribution function, $F(x)$, and the empirical cumulative distribution function $S_n(x)$ defined as

$$S_n(x) = \frac{\text{no. of } x_i \leq x}{n}. \tag{1}$$

The test procedure is summarized as follows: given a sample from some population, the EDF tests reject $H_0{:}F(x) = F_H(x)$ when the difference between $F_H(x)$ and $S_n(x)$ is large. Here $F_H(x)$ is the hypothesized distribution. In general, EDF tests are valid when the distribution is fully specified. However, David and Johnson showed that the distribution of

an EDF statistic depends only on the functional form of the distribution and not on the unknown parameters when the estimated parameters are location and scale (6). It is this principle that permits us to generate valid critical value tables for the gamma distribution which depend only on the shape parameter and sample size.

It is important to note that this thesis uses a modified form of the EDF statistic, because the cumulative distribution is not fully specified. An estimated distribution function is used whose parameters are derived from the observed sample.

The Kolmogorov-Smirnov Statistic

The Kolmogorov-Smirnov (K-S) statistic is defined as the absolute value of the difference between $F(x)$ and $S_n(x)$ or,

$$D = | F(x) - S_n(x) |. \quad (2)$$

In using the K-S statistic for the goodness-of-fit test, we are interested in the greatest absolute difference between $F(x)$ and $S_n(x)$ (2). Therefore the test statistic is

$$T = \sup_x |F(x)-S_n(x)|. \quad (3)$$

The Anderson-Darling Statistic

It is known that goodness-of-fit tests which use actual observations without grouping are sensitive to discrepancies at the tails of the distribution rather than near the median (26:2). The Anderson-Darling test statistic overcomes this problem by accentuating the values of $S_n(x) - F(x)$ where the test statistic is desired to have sensitivity. More

specifically, the Anderson-Darling statistic is based on a weighted average of the squared discrepancy, (i.e. $[S_n(x) - F(x)]^2$ weighted by $\Psi(F(x))$ or

$$A_n^2 = n\int_{-\infty}^{\infty}[S_n(x)-F(x)]^2 \ \Psi(F(x))dF(x), \tag{4}$$

where

$$\Psi(F(x)) = [F(x) \cdot (1-F(x))]^{-1}. \tag{5}$$

Using the computational form

$$A_n^2 = -n - \frac{1}{n} \sum_{j=1}^{n}(2j-1)[\ln F(xj) + \ln (1-F_{n-j+1})], \tag{6}$$

the test procedure is as follows:

1) Let $x_1 \leq x_2 \leq \ldots \leq x_n$ be n observations in the sample.

2) Compute A_n^2

3) If A_n^2 is too large, the hypothesis is to be rejected.

<u>The Cramer-von Mises Statistic</u>

The Cramer-von Mises statistic is a special case of the A_n^2 with $[F(x)] = 1$ and is written as

$$W_n^2 = \int_{-\infty}^{\infty}[S_n(x) - F(x)]^2 \ dx. \tag{7}$$

This test procedure is the same as outlined for the Anderson-Darling goodness-of-fit test (26). The computational form used in this case would be

$$W_n^2 = \frac{1}{12n} + \sum_{j=1}^{n} [F(x_j) - \frac{2j-1}{2n}]^2. \tag{8}$$

<u>Problem Statement</u>

Few goodness-of-fit tests are available to perform on sample data when the parameters of the distribution are not known. As mentioned earlier, Lilliefors developed a test for the unspecified exponential and normal. Also, Bush

generated a set of critical values for the Weibull with unspecified scale and location parameters (3). There still exists the need to develop a valid goodness-of-fit test for the gamma density function when the scale and location parameters are unknown.

The purpose of this research is to develop a goodness-of-fit test for the 3 parameter gamma when the scale and location parameters must be estimated from the sample data. This involves generating a table of critical values based on the sample size and the shape parameter. The accuracy of the critical values must be sufficient enough so that data, sampled from other populations are rejected.

Objectives

This thesis has the following objectives:

1) To generate and document the Anderson-Darling, Cramer-von Mises, and the Kolmogorov-Smirnov rejection tables for the three parameter gamma distribution where the scale and location parameters are unknown.

2) To conduct a power comparison between the Anderson-Darling, Cramer-von Mises, and Kolmogorov-Smirnov goodness-of-fit tests.

3) To investigate the possibilities of a functional relationship between the shape parameter and the critical values in objective one.

II. The Gamma Distribution

The Gamma Density Function

The gamma density function is useful in reliability and maintainability theory. It also has applications in the natural sciences. If the random variable x is gamma distributed then the probability density function takes the form:

$$f(x) = \frac{(x-c)^{K-1}}{\Gamma(K)\,\theta^{K}} \exp\left(-\frac{(x-c)}{\theta}\right), \qquad (9)$$

$$\theta,\ K > 0; \quad x \geq c \geq 0.$$

There are three parameters which specify the gamma. θ is the scale parameter; K is the shape parameter, and c, the location parameter.

The complexity and versatility of the gamma distribution can be observed by examining the graphs of the distribution for various values of the shape and scale parameters. Figure 1a shows graphs of the standardized gamma (i.e., c=0, =1) for K=.5,2,3,4, and 5 (7:370). When K=1, the gamma is the exponential. Also, it is interesting to note that when K is less than one, the gamma closely resembles the exponential distribution.

Also, Figure 1b shows that for large K (K=50), the gamma resembles the normal distribution. Figure 2 illustrates the influence of on the graph of the gamma. Here we set c= and K=2 and sketch the graphs for θ =1/3, 1/2, 1 and 2 (7:371).

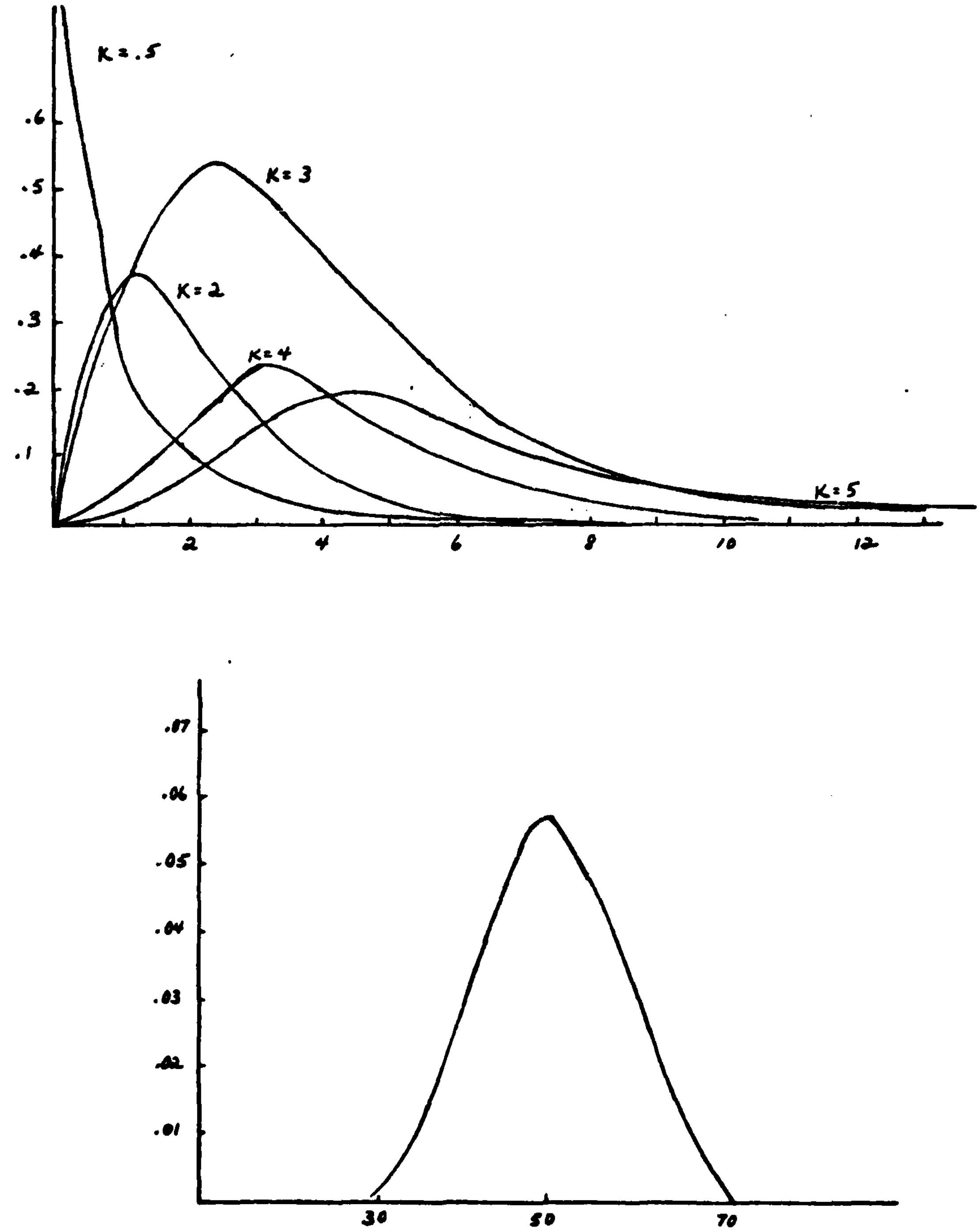

FIG 1. Graphs of the standard gamma density function for (a), K = .5,2,3,4 & 5 and (b) K=50

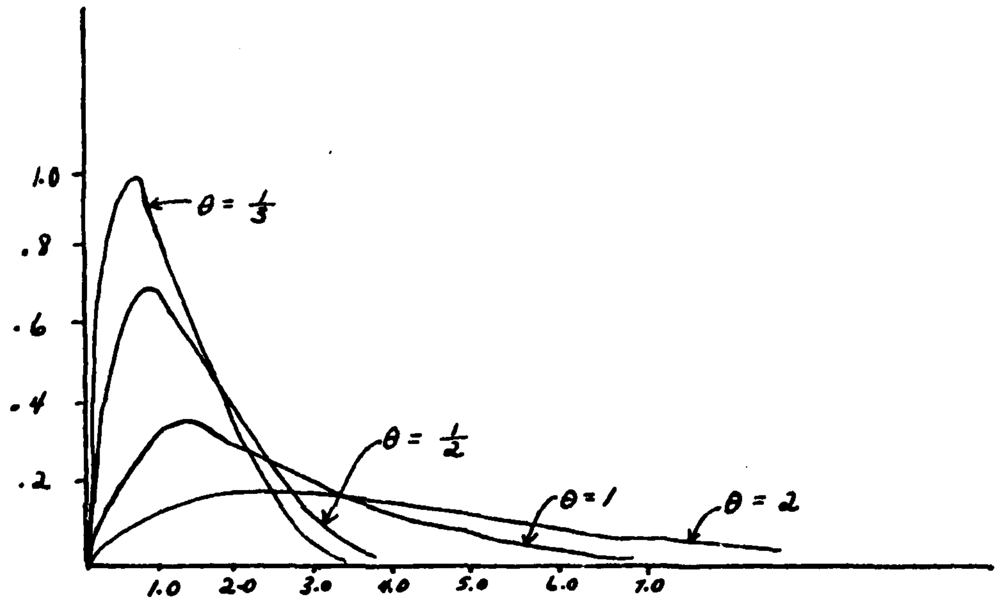

FIG 2. Graphs of the standard gamma density function for $c = 0$, $K = 2$ and $\theta = 1/3, 1/2, 1,$ and 2.

Application of the Gamma Density Function

The gamma distribution is often used in reliability theory to represent the distribution of the time between failures of a system. Assume, for example, that a system is made up of *r* components, all of which must fail for the system to fail. Furthermore, assume that the time to failure x_i of each component is independent and exponentially distributed. Then the time to system failure $Y = X_1 + X_2 + \ldots + X_r$ is gamma distributed (7:369).

In queueing theory, the random variable T follows a gamma distribution, where $T = X_1 + X_2 + \ldots X$ is the total time to service K customers assuming that the time of service of each customer is independent and exponentially distributed.

The two cases described can be modeled as a special case of the gamma known as the Erlang distribution and expressed as:

$$f(t) = \frac{(\ t)^K}{(K-1)!} \cdot t^{K-1} e^{-K\ t}, \quad t \geq 0. \qquad (10)$$

A random variable having a gamma distribution has also been used to represent or measure the occurrence of physical phenomena. For example, Slack and Krubein (1955) demonstrated that the mean value x of radioactivity (alpha particles per minute) within a sample of Pennsylvania shale followed a gamma distribution (7:370).

III. Methodology

This chapter presents the Monte Carlo simulation procedure used in generating the critical value tables for the modified Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling tests. The procedure is outlined using the flow chart in Figure 3. Secondly, an outline of the power comparison among the three goodness-of-fit tests is given. Thirdly, a discussion of the analysis of the functional relationship between the shape parameter and critical values is presented for each of the test statistics.

Monte Carlo Simulation Procedure

The following procedure is used to generate the critical value tables for the modified goodness-of-fit tests. As mentioned earlier Figure 3 presents these steps in flow chart format.

1) For a fixed sample size n and fixed shape parameter K, n standard random gamma deviates are generated using a computer subroutine. The standard gamma deviates are converted to random deviates with location parameter C = 10 and scale parameter $\theta = 1$.

2) The n random deviates are ordered, $x_{(1)}$, $x_{(2)}$, ... $x_{(n)}$.

3) The ordered random deviates are used to estimate the maximum likelihood scale and location parameters.

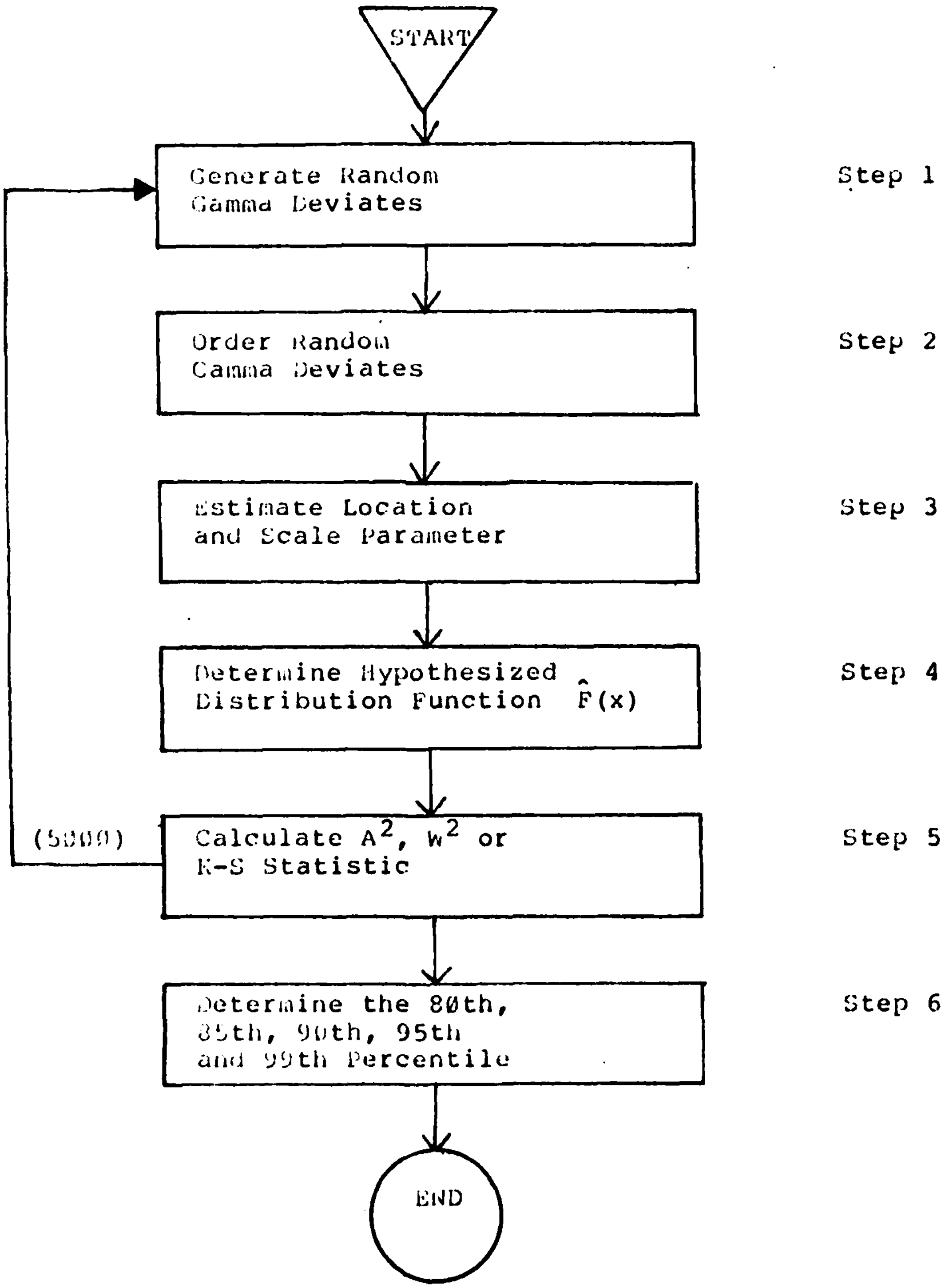

FIG 3. Flow chart

4) The estimated scale and location parameters and fixed shape parameter are used to determine the hypothesized distribution function F(x).

5) The test statistic is calculated using equations three, six, and eight, for the modified Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises tests respectively.

6) Steps one through five are repeated 5000 times.

7) The value of each statistic are ordered in ascending order and the 80th, 85th, 90th, 95th and 99th percentiles are used as the critical values of the test.

Generation of the Three Parameter Gamma Deviates

For the gamma distribution function, there is no closed form for which we could obtain an inverse; however algorithms are available which can be used to generate random gamma deviates. The IMSL subroutine GGMAR is used to generate standard gamma deviates in this thesis. These standard deviates are converted to deviates having location C = 10 and scale θ = 1. This is done by using the transformation

$$z = \theta \cdot x + C \tag{11}$$

where x represents a standard random deviate. This transformation is made to avoid a problem with the parameter estimating routine. Further discussion on this matter is presented in the following section.

Maximum Likelihood Estimates for the Gamma Parameters

The procedure used to calculate the maximum likelihood estimates for gamma parameters was developed by Harter and

Moore (12). Their analysis involves the derivation of the maximum likelihood estimators and includes an iterative method for solving the simultaneous equations. To derive the maximum likelihood equations we begin with the gamma density function with location parameter $C \geq 0$, scale parameter θ, and shape parameter K:

$$f(x,c,\theta,K) = \frac{1}{\Gamma(K)\theta}\left[\frac{x-c}{\theta}\right]^{K-1} \exp\left[-\frac{x-c}{\theta}\right], \tag{12}$$

$$\theta,\ K>0 \quad x \geq C \geq 0.$$

The likelihood function of the order statistics $x_1, x_2, \ldots x_n$ of a sample of size n is

$$L = \left(\frac{1}{\Gamma(k)\theta}\right)^n \sum_{i=1}^{n} \left[\frac{x_i-c}{\theta}\right]^{k-1} \exp\left[-\sum_{i=1}^{n}\frac{x_i-c}{\theta}\right]. \tag{13}$$

We wish to find the values of θ, K, and C which maximize L. This is done by taking the natural logarithm of L, and setting the partial derivatives with respect to the three parameters equal to zero and solving the three simultaneous equations. The partial derivatives are shown here:

$$\frac{\partial \ln L}{\partial \theta} = -\frac{nk}{\theta} + \sum_{i=1}^{n} \frac{x_i-c}{\theta^2} \tag{14}$$

$$\frac{\partial \ln L}{\partial K} = -n\ln\theta + \sum_{i=1}^{n} \ln(x_i-c) - n\,\frac{\partial\Gamma(K)}{\partial K}\,\frac{1}{\Gamma(K)} \tag{15}$$

$$\frac{\partial \ln L}{\partial C} = (1-K) \sum_{i=1}^{n} (x_i-c)^{-1} + \frac{n}{\theta} \tag{16}$$

It should be noted that because of a limitation of the Harter and Moore subroutine, it is not possible to estimate the parameters when the gamma deviates are generated with location $C = 0$. In the event that gamma deviates are

generated with $C = 0$, it is possible to obtain negative estimates for the parameters. The subroutine maps these negative estimates onto zero. Thus, $\hat{C}$ and $\hat{\theta}$ will not retain the invariant property which is needed for these tests to be valid. In addition, the iterative technique used in the Carter and Moore subroutine does not work for the special case when the shape parameter is set to one. Because the gamma is an exponential distribution when the shape parameter is equal to one, we can use the maximum likelihood estimators of the location and scale parameters for the exponential. Therefore, setting K equal to one and solving equations 13 and 15 we obtain

$$C = x_{(1)} \tag{17}$$

and

$$\theta = \frac{1}{n} \sum_{i=1}^{n} x_i - x_{(1)} \tag{18}$$

<u>Deriving the Hypothesized Distribution Function $\hat{F}(x)$</u>

The maximum likelihood estimates for the location and scale parameters and the fixed shape parameter determine $\hat{F}(x)$. Obtaining a numerical value for $\hat{F}(x)$ requires an integral calculation; this calculation was done using the IMSL subroutine MDGAM (13). MDGAM calculates the probability that a random variable x from a standard gamma distribution (i.e., $C = 0$ and $\theta = 1$) is less than or equal to x. To transform the deviates, y_i, from a generalized gamma distribution into standard gamma deviates we use

$$x_i = \frac{y_i - \hat{C}}{\hat{\theta}}. \tag{19}$$

The details of this transformation are provided in Cortes (5:17).

Power Comparison

The powers of the modified Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling tests are compared for ten alternative distributions. Samples of sizes equal to five, 15, and 25 are drawn from the following selected distributions:

1) Gamma, shape equals 1.5
2) Gamma, shape equals 2.5
3) Gamma, shape equals 4.0
4) Weibull, shape equals 2.0
5) Weibull, shape equals 3.0
6) Normal (10,1)
7) Beta ($p = 1$, $q = 2$)
8) Beta ($p = 2$, $q = 2$)
9) Lognormal ($p = 1$, $w = 0$)
10) Lognormal ($p = 2$, $w = 0$)

These distributions are tested according to:

H_0: The sample variates follow a gamma distribution having shape parameter K.

H_A: The sample variates follow some other distribution.

The power investigation was conducted under two null hypotheses, one for shape parameter, $K = 1.5$, the other for shape parameter, $K = 4.0$. The random deviates for the above alternative distributions were generated using IMSL subroutines.

The lognormal density function is written as

$$f(x) = \frac{1}{x\rho\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left[\frac{\ln x-\omega}{\rho}\right]^2\right), \quad x>0 \qquad (20)$$

$$= 0 \text{ otherwise.}$$

and is illustrated in Figures 4a, and 4b for the parameters ρ and ω given above.

The Weibull density function is

$$f(x) = \frac{K(x-c)^{K-1}}{\theta^K} \exp\left[-\frac{(x-c)}{\theta}\right]^K, \quad K, \theta > 0, \ C \leq x \qquad (21)$$

$$= 0 \text{ otherwise.}$$

and is shown in Figures 5a and 5b.

The beta density function is expressed as

$$f(x) = \frac{\Gamma(p+q)}{\Gamma(p)\,\Gamma(q)} x^{p-1}(1-x)^{q-1}, \quad 0<x<1 \qquad (22)$$

$$= 0\text{, otherwise.}$$

and its graph for the two cases of interest, is presented in Figures 6a and 6b.

For each of the sample sizes mentioned above, five thousand sample sets were generated for the alternative distributions. The location and scale parameters are calculated under the null hypothesis and the three test statistics, K-S, A^2, and W^2 are evaluated. The value of these statistics are compared to the critical values derived in this thesis. If the value of the statistic is greater than the critical value, the null hypothesis is rejected. The

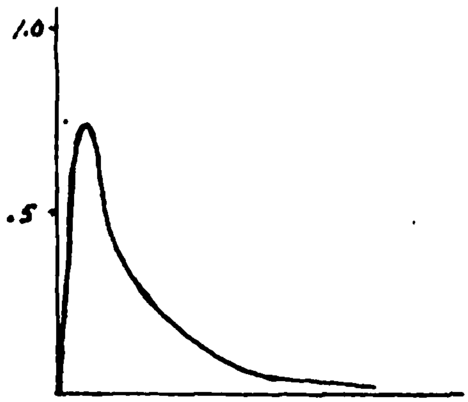

FIG 4a. Lognormal
w = 0. p = 1.

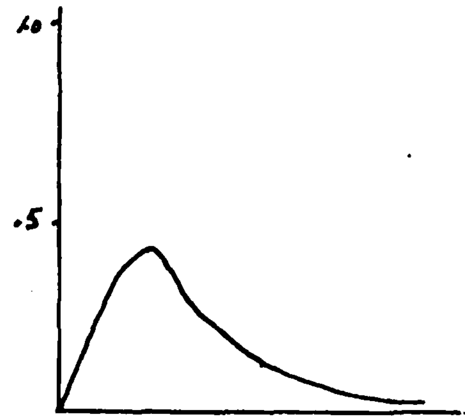

FIG 4b. Lognormal
w = 0. p = 2.

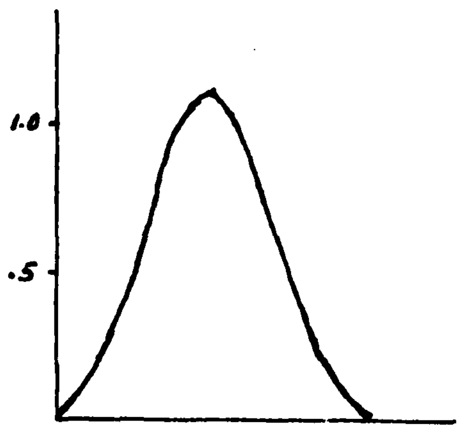

FIG 5a. Weibull, K = 3.
θ = 1. C = 0.

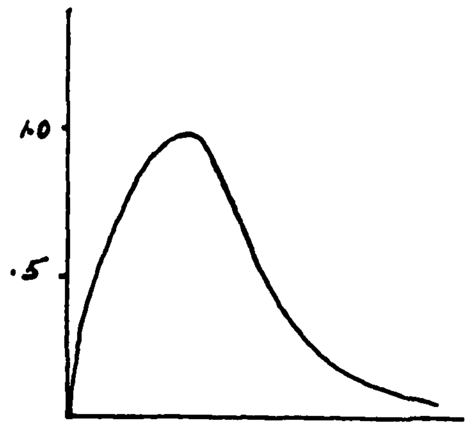

FIG 5b. Weibull, K = 2.0.
θ = 1. C = 0.

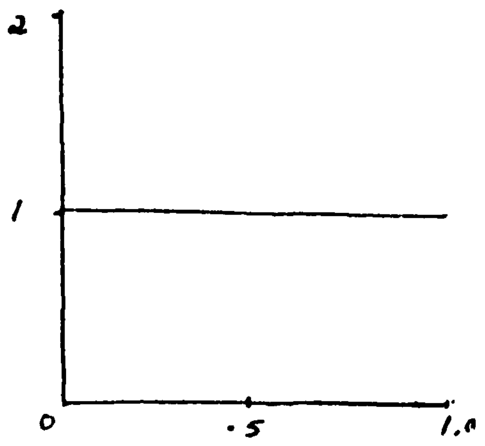

FIG 6a. Beta,
p = 1. q = 1.

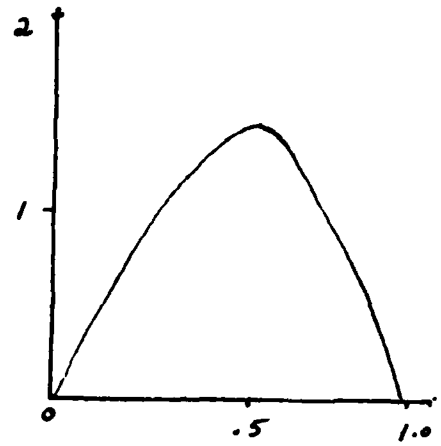

FIG 6b. Beta,
p = 2. q = 2.

total number of rejections are counted. The power is the total number of rejections divided by the number of trials, 5000.

Determining the Critical Values

By repeating steps one through five 5000 times as shown in the flow chart in Figure 7, 5000 values for K-S, A^2, and W^2 are calculated. These critical values are ordered and the 80th, 85th, 90th, 95th, and 99th percentile are used as the critical values of the tests.

Computer Programs

The computer programs used in this thesis are presented in Appendix C.

IV. Use of Tables

In this chapter a set of steps is given which is used to perform a goodness-of-fit test by applying any of the three tests developed in this thesis. Also, an example illustrating the Anderson-Darling test is presented.

The folowing steps are used to perform a goodness-of-fit test:

1) Determine the shape parameter, K, and the desired level of significance α.

2) From the data to be tested, calculate the maximum likelihood estimators for the location and scale parameters.

3) From the appropriate table, select the critical value, $d_{\alpha,n}$ corresponding to α, the sample size n, and shape parameter K.

4) Using the maximum likelihood estimators, determine the estimated hypothesized distribution, and use equation three, six, or eight to calculate the Kolmogorov-Smirnov, Anderson-Darling, or Cramer-von Mises test statistic respectively.

5) If the value obtained in step four is greater than the critical value found in step three, then reject the hypothesized distribution. If it is smaller than the critical value, then the hypothesized distribution can not be rejected.

Example

The time between failures of a particular subsystem of a radar system is believed to be distributed according to a gamma distribution with shape parameter equal to 3.0. A test engineer recorded the following times between failures of that subsystem: 11.1, 10.6, 10.4, 13.0, 11.3, 10.5, 10.6, 10.9, 10.6, 10.8 days.

A modified Anderson-Darling test at a .05 level of significance is performed using the critical values in this thesis. The problem can be stated as a test of hypothesis, that is,

H_0: The distribution is gamma (shape = 3.0).

H_A: The sample comes from another distribution.

First, the level of significance α, and shape parameter K, have been determined to be .05 and 3.0 respectively. Second, the maximum likelihood estimators calculated using the Harter and Moore subroutine are $\hat{c}$ = 9.319 and $\hat{\theta}$ = .826. Next, the critical value from Table XXVI is .8415. The hypothesized distribution is completely determined by the fixed shape parameter and the estimated location and scale parameters; these values are presented in Table I. The value of the Anderson-Darling statistic is A^2 = 1.7342. Since 1.7342 is greater than .8415, the null hypothesis is rejected. Therefore, the conclusion is that the sample of time between failures comes from some other distribution.

TABLE I

Example: x, F(x)

i	x	$\hat{F}(x)$
1	10.4	.153
2	10.5	.163
3	10.6	.203
4	10.6	.211
5	10.8	.269
6	10.9	.300
7	11.1	.384
8	11.3	.432
9	13.0	.822
10	18.6	.999

V. Discussion of the Results

This chapter presents the results obtained with respect to the objectives stated in chapter 1. These objectives were to develop modified Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises tests for the gamma and compare their powers. Also included was an investigation of the relationship between the critical values of each test and shape parameters. Included with these results is a report on the validation of the computer programs used in this thesis.

Presentation of the Kolmogorov-Smirnov, Cramer-von Mises, and Anderson-Darling Tables of Critical Values

The tables of critical values for the modified K-S, A^2, and W^2 tests are presented in Appendices A, B, and C, respectively.

When the shape parameter is fixed, both the K-S and A^2 critical values are decreasing as the sample size increases. The rate of decrease is smaller as n increases; this is an indication that the critical values appear to be converging for large sample sizes. The Cramer-von Mises critical values, on the other hand, are increasing with respect to the sample size. Again, the rate of increase is smaller for larger values of n, indicating that the critical values are converging for large sample sizes.

It should be noted that because the critical values are derived through Monte Carlo simulations, the values are not error free and that the amount of error decreases as the number of trials increases (25). The 5000 repetitions used

in this thesis was a practical compromise based on computer time required and accuracy desired.

Computer Programs Validation

The computer programs are verified by generating critical values for the exponential distribution with unknown mean. This is done by generating gamma deviates with shape parameter equal to one, fixing the location parameter at some arbitrary value, and estimating only the scale parameter .

The critical values are calculated for sample sizes n = 5, 10, 20, and 30. The critical values from the Anderson-Darling and Cramer-von Mises statistics are modified using expressions (23) and (24) derived by Stephens (27):

$$A^2 \left(1 + \frac{1.5}{n} - \frac{5}{n^2}\right) \tag{23}$$

$$W^2 \left(1 + \frac{.16}{n}\right). \tag{24}$$

The computed critical values are compared to those calculated by Stephens (27) for significance levels .15,

The critical values calculated for the Kolmogorov-Smirnov statistic for the exponential are compared directly to those derived by Lilliefors (28). The critical values which are derived from the programs in this thesis are presented in Table IV and can be compared to Lilliefors results in Table V.

The critical values for the Kolmogorov-Smirnov statistic compared very well to the Lilliefors values. The

TABLE II

Cramer-von Mises W^2

1-x	W^2 (1 + .16/n)				Stephen's Critical Values
	n=5	n=10	n=20	n=30	
.85	.148	.149	.148	.1523	.149
.90	.177	.176	.174	.170	.171
.95	.228	.219	.221	.221	.224
.99	.341	.332	.335	.325	.337

TABLE III

Anderson-Darling A^2

1-x	A^2 (1 + 1.5/n - 5/n^2)				Stephen's Critical Values
	n=5	n=10	n=20	n=30	
.85	.941	.992	.953	.961	.922
.90	1.099	1.129	1.097	1.098	1.078
.95	1.397	1.381	1.376	1.372	1.341
.99	2.070	2.219	2.077	1.974	1.957

TABLE IV

Kolmogorov-Smirnov
Thesis Critical Values

1-	n=5	n=10	n=20	n=30
.85	.378	.276	.199	.166
.90	.401	.295	.214	.178
.95	.447	.328	.235	.193
.99	.531	.384	.278	.232

TABLE V

Kolmogorov-Smirnov K-S
Lilliefors Critical Values

1-	n=5	n=10	n=20	n=30
.85	.382	.277	.199	.164
.90	.406	.295	.212	.174
.95	.442	.325	.234	.192
.99	.504	.380	.278	.226

greatest deviation occurs for n = 5 at significance level .01. The Cramer-von Mises values generated are very close to Stephens values with the greatest deviation being 3.6% for n = 30 and significance level at .01. There was also a good match between the Anderson-Darling values, with most deviations between 3% and 4%; however the greatest deviation is 13.4% for n = 10 and significance level .01.

<u>Power Investigation</u>

A power comparison is made between the Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises goodness-of-fit tests developed in this thesis. The gamma distribution with shape parameters equal to 1.5 and 4.0 were both used against the alternative distributions listed in chapter III. Sample sizes five, 15, and 25 were used in the power studies at both an α-level of .05 and .01.

Tables VI through XIII show the results of the power comparisons for α-levels .05 and .01. When the null hypothesis is true, the power meets the claimed level of significance to the second decimal place in most cases. For all tests the power is low for sample sizes equal to five. In fact, in most cases for n = 5 the power is nearly equal to the significance of the test, indicating that the goodness-of-fit test has no practical use for very small sample sizes.

In nearly all cases, the powers of the Cramer-von Mises and/or Anderson-Darling are greater than Kolmogorov-Smirnov tests. Based on this study, the latter test would not be

TABLE VI

Power Test for the Gamma Distribution
H_0: Gamma Distribution, α = 4.0 -- H_a: Another Distribution
Level of Significance = .05

Sample Size n	Test Statistics	Alternative Distributions Gamma Shape=1.5	Gamma Shape=2.5	Gamma Shape=4.0	Weibull Shape=2.0	Weibull Shape=3.0
25		K-S=.177 W^2 =.202 A^2 =.226	K-S=.072 W^2 =.075 A^2 =.075	K-S=.052 W^2 =.051 A^2 =.048	K-S=.063 W^2 =.063 A^2 =.054	K-S=.198 W^2 =.230 A^2 =.218
15		K-S=.117 W^2 =.135 A^2 =.147	K-S=.060 W^2 =.062 A^2 =.064	K-S=.052 W^2 =.051 A^2 =.045	K-S=.054 W^2 =.057 A^2 =.050	K-S=.116 W^2 =.133 A^2 =.119
5		K-S=.075 W^2 =.076 A^2 =.078	K-S=.061 W^2 =.061 A^2 =.063	K-S=.057 W^2 =.055 A^2 =.054	K-S=.047 W^2 =.045 A^2 =.042	K-S=.050 W^2 =.050 A^2 =.040

TABLE VI Continued

Power Test for the Gamma Distribution
H_0: Gamma Distribution, K = 4.0 -- H_a: Another Distribution
Level of Significance = .05

Sample Size n	Test Statistics	Alternative Distributions				
		Normal (10,1)	Lognormal ω=0 ρ=1	Lognormal ω=0 ρ=2	Beta p=1 q=1	Beta p=2 q=2
25		K-S=.155 W^2 =.375 A^2 =.368	K-S=.643 W^2 =.728 A^2 =.761	K-S=.993 W^2 =.997 A^2 =.999	K-S=.19[illegible] W^2 =.2[illegible]9 A^2 =.29[illegible]	K-S=.058 W^2 =.211 A^2 =.204
15		K-S=.173 W^2 =.210 A^2 =.197	K-S=.424 W^2 =.491 A^2 =.517	K-S=.915 W^2 =.949 A^2 =.963	K-S=.129 W^2 =.162 A^2 =.103	K-S=.112 W^2 =.131 A^2 =.117
5		K-S=.057 W^2 =.058 A^2 =.047	K-S=.150 W^2 =.159 A^2 =.175	K-S=.439 W^2 =.462 A^2 =.489	K-S=.061 W^2 =.065 A^2 =.053	K-S=.055 W^2 =.054 A^2 =.044

TABLE VII

Power Test for the Gamma Distribution
H_0: Gamma Distribution, K = 1.5 -- H_a: Another Distribution
Level of Significance = .05

Sample Size n	Test Statistics	Alternative Distributions: Gamma Shape=1.5	Gamma Shape=2.5	Gamma Shape=4.0	Weibull Shape=2.0	Weibull Shape=3.0
25		K-S=.057 W^2 =.051 A^2 =.044	K-S=.0968 W^2 =.0994 A^2 =.0704	K-S=.197 W^2 =.228 A^2 =.175	K-S=.275 W^2 =.313 A^2 =.247	K-S=.607 W^2 =.685 A^2 =.628
15		K-S=.0458 W^2 =.047 A^2 =.0448	K-S=.053 W^2 =.056 A^2 =.037	K-S=.093 W^2 =.101 A^2 =.069	K-S=.123 W^2 =.145 A^2 =.093	K-S=.290 W^2 =.351 A^2 =.280
5		K-S=.051 W^2 =.047 A^2 =.049	K-S=.044 W^2 =.036 A^2 =.0302	K-S=.0422 W^2 =.034 A^2 =.026	K-S=.034 W^2 =.028 A^2 =.017	K-S=.040 W^2 =.030 A^2 =.015

TABLE VII Continued

Power Test for the Gamma Distribution

H_0: Gamma Distribution, K = 1.5 -- H_a: Another Distribution

Level of Significance = .05

Sample Size n	Test Statistics	Alternative Distributions: Normal (10,1)	Lognormal $\omega=0$ $\rho=1$	Lognormal $\omega=0$ $\rho=2$	Beta p=1 q=1	Beta p=2 q=2
25		K-S=.726 W^2 =.797 A^2 =.743	K-S=.390 W^2 =.442 A^2 =.444	K-S=.985 W^2 =.991 A^2 =.995	K-S=.361 W^2 =.400 A^2 =.413	K-S=.515 W^2 =.636 A^2 =.566
15		K-S=.387 W^2 =.453 A^2 =.377	K-S=.252 W^2 =.295 A^2 =.307	K-S=.873 W^2 =.909 A^2 =.928	K-S=.181 W^2 =.226 A^2 =.184	K-S=.245 W^2 =.304 A^2 =.234
5		K-S=.050 W^2 =.038 A^2 =.015	K-S=.114 W^2 =.118 A^2 =.122	K-S=.383 W^2 =.405 A^2 =.423	K-S=.047 W^2 =.041 A^2 =.020	K-S=.045 W^2 =.034 A^2 =.017

TABLE VIII

Power Test for the Gamma Distribution

H_0: Gamma Distribution, K = 4.0 -- H_a: Another Distribution

Level of Significance = .01

Sample Size n	Test Statistics	Alternative Distributions Gamma Shape=1.5	Gamma Shape=2.5	Gamma Shape=4.0	Weibull Shape=2.0	Weibull Shape=3.0
25		K-S=.057 W^2 =.066 A^2 =.079	K-S=.017 W^2 =.015 A^2 =.017	K-S=.010 W^2 =.009 A^2 =.008	K-S=.012 W^2 =.011 A^2 =.008	K-S=.061 W^2 =.082 A^2 =.074
15		K-S=.034 W^2 =.04 A^2 =.049	K-S=.013 W^2 =.015 A^2 =.017	K-S=.010 W^2 =.010 A^2 =.009	K-S=.012 W^2 =.013 A^2 =.010	K-S=.036 W^2 =.043 A^2 =.037
5		K-S=.018 W^2 =.019 A^2 =.024	K-S=.015 W^2 =.014 A^2 =.016	K-S=.014 W^2 =.013 A^2 =.012	K-S=.011 W^2 =.010 A^2 =.007	K-S=.011 W^2 =.009 A^2 =.007

TABLE VIII Continued

Power Test for the Gamma Distribution
H_0: Gamma Distribution, K = 4.0 -- H_a: Another Distribution
Level of Significance = .01

Sample Size n	Test Statistics	Alternative Distributions				
		Normal (10,1)	Lognormal $\omega=0$ $\rho=1$	Lognormal $\omega=0$ $\rho=2$	Beta p=1 q=1	Beta p=2 q=2
25		K-S=.141 W^2 =.193 A^2 =.187	K-S=.427 W^2 =.526 A^2 =.558	K-S=.968 W^2 =.985 A^2 =.991	K-S=.028 W^2 =.075 A^2 =.075	K-S=.051 W^2 =.063 A^2 =.054
15		K-S=.068 W^2 =.088 A^2 =.083	K-S=.257 W^2 =.315 A^2 =.355	K-S=.917 W^2 =.883 A^2 =.957	K-S=.035 W^2 =.038 A^2 =.037	K-S=.034 W^2 =.040 A^2 =.034
5		K-S=.013 W^2 =.011 A^2 =.009	K-S=.056 W^2 =.066 A^2 =.077	K-S=.269 W^2 =.306 A^2 =.340	K-S=.012 W^2 =.012 A^2 =.008	K-S=.008 W^2 =.008 A^2 =.006

TABLE IX

Power Test for the Gamma Distribution
H_0: Gamma Distribution, K = 1.5 -- H_a: Another Distribution
Level of Significance = .01

Sample Size n	Test Statistics	Alternative Distributions				
		Gamma Shape=1.5	Gamma Shape=2.5	Gamma Shape=4.0	Weibull Shape=2.0	Weibull Shape=3.0
25		K-S=.011 W^2 =.010 A^2 =.011	K-S=.027 W^2 =.022 A^2 =.015	K-S=.073 W^2 =.080 A^2 =.064	K-S=.110 W^2 =.122 A^2 =.102	K-S=.358 W^2 =.443 A^2 =.403
15		K-S=.007 W^2 =.008 A^2 =.009	K-S=.009 W^2 =.009 A^2 =.004	K-S=.022 W^2 =.024 A^2 =.011	K-S=.033 W^2 =.036 A^2 =.020	K-S=.106 W^2 =.142 A^2 =.086
5		K-S=.010 W^2 =.013 A^2 =.011	K-S=.007 W^2 =.007 A^2 =.006	K-S=.006 W^2 =.006 A^2 =.005	K-S=.003 W^2 =.002 A^2 =.002	K-S=.003 W^2 =.001 A^2 =.001

TABLE IX Continued

Power Test for the Gamma Distribution

H_0: Gamma Distribution, $K = 1.5$ -- H_a: Another Distribution

Level of Significance = .01

Sample Size n	Test Statistics	Alternative Distributions				
		Normal (10,1)	Lognormal $\omega=0$ $\rho=1$	Lognormal $\omega=0$ $\rho=2$	Beta $p=1$ $q=1$	Beta $p=2$ $q=2$
25		K-S=.515 W^2 =.600 A^2 =.567	K-S=.230 W^2 =.268 A^2 =.293	K-S=.952 W^2 =.969 A^2 =.991	K-S=.135 W^2 =.188 A^2 =.166	K-S=.264 W^2 =.338 A^2 =.302
15		K-S=.181 W^2 =.228 A^2 =.164	K-S=.127 W^2 =.164 A^2 =.167	K-S=.748 W^2 =.813 A^2 =.844	K-S=.050 W^2 =.062 A^2 =.038	K-S=.086 W^2 =.108 A^2 =.066
5		K-S=.004 W^2 =.001 A^2 =.001	K-S=.048 W^2 =.049 A^2 =.047	K-S=.213 W^2 =.244 A^2 =.240	K-S=.004 W^2 =.002 A^2 =.002	K-S=.003 W^2 =.002 A^2 =.001

TABLE X

Coefficients and R^2 Values for the Relationships between the Kolmogorov-Smirnov Critical Values and the Gamma Shape Parameters, 1.0 (.5) 4.0

Level of Significance	.20	.15	.10	.05	.01
n= 5	.0505 .3116 .98	.0542 .3273 .98	.0672 .3452 .97	.1051 .3685 .98	.1138 .4215 .99
n=10	.0512 .2281 .97	.0573 .2387 .95	.0650 .2527 .95	.0679 .2762 .96	.0894 .3190 .95
n=15	.0447 .1886 .99	.0499 .1973 .97	.0559 .2092 .98	.0636 .2206	.0785 .2654 .96

Legend a_1 R^2 a_0

TABLE X, Continued

Level of Significance	.20	.15	.10	.05	.01
n=20	.0417 .1653 .98	.0456 .1731 .98	.0509 1837 .99	.0578 .1998 .95	.0862 .2297 .95
n=25	.0325 .1490 .94	.0367 .1562 .94	.0373 .1659 .92	.0420 .1809 .92	.0537 .2098 .91
n=30	.0317 .1363 .97	.0349 .1432 .96	.3691 .1526 .94	.0420 .1663 .95	.0639 .1937 .98
Legend a_1 R^2 a_2					

TABLE X, Continued

Level of Significance	.20	.15	.10	.05	.01
n=20	.0417 .1[illegible]53 .98	.0456 .1731 .98	.0509 .1837 .99	.0570 .1998 .95	.0862 .2297 .95
n=25	.0325 .1490 .94	.0367 .1562 .94	.0373 .1659 .92	.0420 .1809 .92	.0537 .2398 .91
n=30	.0317 .1363 .97	.0349 .1432 .95	.3591 .1526 .94	.0420 .1663 .95	.0639 .1937 .98

Legend a_1
 a_0 R^2

TABLE XI, Continued

Level of Significance	.20	.15	.10	.05	.01
n=20	.4124 .5350 .99	.4707 .5861 .99	.5789 .6565 .99	.6945 .7388 .99	1.2195 1.0328 .99
n=25	.4613 .5241 .99	.5048 .5767 .99	.5840 .6553 .99	.7675 .7860 .99	1.0620 1.0911 .94
n=30	.4560 .5293 .99	.5175 .5819 .99	.6169 .6554 .99	.7834 .7810 .98	1.2103 1.0977 .96
Legend a_1 R^2 a_0					

TABLE XII

Coefficients and R^2 Values for the Relationships between the Cramer-von Mises Critical Values and the Gamma Shape Parameters, 1.0 (.5) 4.0

Level of Significance	.20	.15	.10	.05	.01
n= 5	.0510 .99 .0871	.0589 .99 .0956	.0739 .99 .1066	.1107 .99 .1238	.1523 .98 .1693
n=10	.0616 .98 .0871	.0715 .99 .0961	.0857 .98 .1104	.1137 .99 .1321	.1766 .97 .1861
n=15	.0584 .99 .0884	.0694 .99 .0917	.0913 .99 .1107	.1146 .99 .1341	.2043 .99 .1857
Legend a_1 R^2 a_0					

TABLE XII, Continued

Level of Significance	.20	.15	.10	.05	.01
n=20	.0748 .0867 .99	.0860 .0965 .99	.1005 .1104 .99	.1264 .1357 .99	.2365 .1836 .98
n=25	.0658 .0871 .99	.0762 .0973 .98	.0856 .1138 .99	.1098 .1379 .97	.1895 .1957 .98
n=30	.0673 .0862 .98	.0826 .0974 .96	.0946 .1124 .95	.1221 .1361 .94	.2203 .1937 .95
Legend	a_1 a_0 R^2				

used as long as the first two are available. An examination of Tables VI through XIII reveals that for both levels of significance, the Cramer-von Mises test is more powerful than the Anderson-Darling test. The only exception to the previous statement occurs when the alternative distribution is lognormal.

The following observations are made concerning the power of both the Cramer-von Mises and Anderson-Darling tests when sample sizes are either 15 or 25:

1) If the null hypothesis is a gamma with shape equal to 1.5, the power is high against all alternative distributions except for another gamma. The tests are especially high against the lognormal distribution.

2) When the null hypothesis is the gamma with shape equal to four, the power is high against the lognormal and normal distributions. The power is not quite so high against the beta. Against the other gammas and the Weibull with shpae equal to two, the power is low, even when the sample size is 25.

Relationship between Critical Values and Shape Parameters

An investigation of the relationship between the shape parameter and the Kolmogorov-Smirnov, Anderson-Darling, and Cramer-von Mises critical values is summarized in this section. For each test statistic, the shape parameter versus critical values are plotted. These graphs are presented in Appendix D for the K-S, Appendix E for A^2, and Appendix F for the W^2 critical values. The graphs of all the test

statistics appear to exhibit a common and consistent behavior. This consistent behavior observed from the graphical representations can be summed up as follows:

1) For all test statistics, the relationship of critical values as a function of shape is always decreasing for shape greater than one; this decrease appears to be an inverse relationship.

2) The graphs of the Anderson-Darling critical values always show an increase as the shape increases from .5 to 1.0.

3) The Kolmogorov-Smirnov and Cramer-von Mises critical values increase or decrease, as the shape varies from .5 to 1.0, depending on the sample size.

A regression analysis is performed to determine the functional relationship between the critical values and shape parameters, as suggested by the graphs. The study includes values for the shape between 1.5 and 4.0. Shape parameters less than 1.5 are not considered in the regression analysis because a different estimating technique was used to calculate the critical values for the gamma when shape is equal to one. In addition, more information is needed about the behavior of the function between .5 and 1.5 for all test statistics.

The expression which best represents the relationship for all test statistics is

$$C = a_0 + a_1(1/K^2).$$

K^2, the number which measured the amount of variation

in all cases. This expression can be used to find the critical value corresponding to shape parameters between 1.5 and 4.0, not found in the tables in Appendices A, B, and C. Therefore, a test of hypothesis can be performed for the null hypothesis being, for example, a gamma distribution having shape equal to 2.75.

The value of R^2, and coefficients a_0 and a_1 are recorded in Tables XIV, XV, and XVI, for the K-S, A^2, and W^2 critical values respectively.

VI. Conclusions and Recommendations

Conclusions

Based on results obtained in this thesis, the following conclusions are noted:

1) The Kolmogorov-Smirnov, Anderson-Darling and Cramer-von Mises critical values for the three-parameter gamma are valid. The power study revealed that when the null hypothesis is true all three tests achieve the claimed level of significance.

2) The power comparison study based on the ten alternative distributions listed in Chapter 3 shows that in general the powers in decreasing order are W^2, A^2, and K-S. The A^2, however is more powerful against the lognormal distribution. All three tests demonstrated low power for sample sizes equal to five, indicating a goodness-of-fit test involving a sample size of five using the tabled critical values would not be practical.

Recommendations

The following recommendations are suggested for further investigation:

1) Develop a more efficient technique to calculate the maximum likelihood estimators for the parameters of a gamma distribution.

2) Investigate a functional relationship between the sample size and critical values so that goodness-of-fit tests can be done for sample sizes other than those presented in this thesis.

3) Extend the goodness-of-fit test to include parameters between zero and one.

4) Examine the feasibility of developing a goodness-of-fit test for distributions whose parameters are unknown, based on the characteristic function.

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APPENDIX A

Tables of the Kolmogorov-Smirnov Critical Values for the Gamma Distribution

TABLE XIII

Kolmogorov-Smirnov
Shape Parameter = .5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3528	.3681	.3913	.4330	.5247
10	.2637	.2798	.3010	.3331	.3862
15	.2249	.2365	.2514	.2804	.3317
20	.1967	.2077	.2225	.2457	.2941
25	.1759	.1870	.2006	.2227	.2623
30	.1635	.1736	.1844	.2039	.2449

TABLE XIV

Kolmogorov-Smirnov
Shape Parameter = 1.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3701	.3848	.3994	.4433	.5269
10	.2651	.2788	.2958	.3267	.3930
15	.2184	.2306	.2451	.2709	.3208
20	.1923	.2038	.2177	.2392	.2774
25	.1693	.1788	.1910	.2113	.2520
30	.1561	.1650	.1759	.1958	.2297

TABLE XV

Kolmogorov-Smirnov
Shape Parameter = 1.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3333	.3505	.3738	.4151	.4725
10	.2496	.2624	.2797	.3046	.3597
15	.2080	.2187	.2336	.2554	.2995
20	.1841	.1935	.2058	.2244	.2658
25	.1624	.1713	.1812	.1979	.2320
30	.1507	.1579	.1677	.1839	.2222

TABLE XVI

Kolmogorov-Smirnov
Shape Parameter = 2.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3255	.3423	.3635	.3927	.4482
10	.2432	.2567	.2730	.2966	.3377
15	.2005	.2105	.2235	.2464	.2845
20	.1748	.1838	.1976	.2157	.2488
25	.1591	.1671	.1773	.1943	.2264
30	.1463	.1537	.1640	.1793	.2099

TABLE XVII

Kolmogorov-Smirnov
Shape Parameter = 2.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3198	.3366	.3581	.3898	.4399
10	.2370	.2483	.2634	.2884	.3378
15	.1963	.2067	.2187	.2414	.2814
20	.1718	.1803	.1911	.2102	.2457
25	.1552	.1636	.1738	.1890	.2209
30	.1417	.1483	.1584	.1730	.2023

TABLE XVIII

Kolmogorov-Smirnov
Shape Parameter = 3.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3179	.3342	.3524	.3799	.4365
10	.2338	.2449	.2595	.2829	.3282
15	.1939	.2041	.2172	.2352	.2751
20	.1710	.1792	.1898	.2067	.2395
25	.1520	.1600	.1695	.1858	.2130
30	.1411	.1478	.1573	.1711	.2026

TABLE XIX

Kolmogorov-Smirnov
Shape Parameter = 3.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3161	.3317	.3508	.3769	.4314
10	.2310	.2421	.2561	.2812	.3234
15	.1923	.2001	.2127	.2326	.2724
20	.1695	.1775	.1884	.2059	.2403
25	.1511	.1588	.1689	.1829	.2127
30	.1382	.1452	.1543	.1684	.1992

TABLE XX

Kolmogorov-Smirnov
Shape Parameter = 4.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.3131	.3289	.3471	.3731	.4266
10	.2308	.2416	.2567	.2792	.3267
15	.1904	.1992	.2116	.2312	.2666
20	.1667	.1748	.1858	.2002	.2312
25	.1505	.1570	.1666	.1823	.2137
30	.1381	.1450	.1544	.1690	.1970

APPENDIX B

Tables of the Anderson-Darling Critical Values for the Gamma Distribution

TABLE XXI

Anderson-Darling
Shape Parameter = .5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	1.0724	1.1679	1.3135	1.6238	2.5899
10	.9224	1.0295	1.1919	1.4375	2.2323
15	.8797	.9824	1.1258	1.4083	2.2116
20	.8704	.9759	1.1352	1.4831	2.1989
25	.8367	.9472	1.0963	1.3910	2.0626
30	.8332	.9428	1.1107	1.4316	2.3048

TABLE XXII

Anderson-Darling
Shape Parameter = 1.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	2.0880	2.3079	2.5926	3.0710	4.3921
10	1.5085	1.6604	1.8807	2.2980	3.4906
15	1.2827	1.4396	1.6493	2.0360	2.9562
20	1.2124	1.3510	1.5480	1.9038	2.7789
25	1.0831	1.2147	1.3854	1.7118	2.5678
30	1.0558	1.1750	1.3756	1.6894	2.4482

TABLE XXIII

Anderson-Darling
Shape Parameter = 1.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.6908	.7577	.8491	1.0603	1.4300
10	.7104	.7832	.9017	1.0920	1.5597
15	.7063	.7940	.9007	1.1067	1.5828
20	.7145	.7903	.9120	1.0923	1.5719
25	.7278	.8053	.9170	1.1332	1.5569
30	.7296	.8105	.9266	1.1189	1.6133

TABLE XXIV

Anderson-Darling
Shape Parameter = 2.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.6106	.6666	.7417	.8673	1.1757
10	.6265	.6949	.7879	.9240	1.2260
15	.6330	.6927	.7880	.9465	1.3451
20	.6422	.7101	.8051	.9592	1.3437
25	.6429	.7002	.7984	.9670	1.3956
30	.6481	.7167	.8213	.9976	1.4640

TABLE XXV

Anderson-Darling
Shape Parameter = 2.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.5700	.6261	.6964	.8324	1.0970
10	.5891	.6504	.7409	.8740	1.2080
15	.5915	.6556	.7330	.8693	1.2046
20	.6109	.6699	.7504	.9029	1.2276
25	.5991	.6590	.7462	.9023	1.2328
30	.6011	.6573	.7394	.9025	1.2614

TABLE XXVI

Anderson-Darling
Shape Parameter = 3.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.5549	.5962	.6730	.7813	1.0634
10	.5687	.6276	.7056	.8415	1.1702
15	.5757	.6376	.7251	.8615	1.1651
20	.5749	.6360	.7164	.8755	1.1499
25	.5708	.6364	.7225	.8808	1.1714
30	.5852	.6490	.7355	.8828	1.2601

TABLE XXVII

Anderson-Darling
Shape Parameter = 3.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.5472	.5967	.6588	.7748	1.0173
10	.5522	.6024	.6730	.8072	1.1318
15	.5577	.6209	.6942	.8081	1.1150
20	.5686	.6259	.7060	.8432	1.1503
25	.5603	.6190	.6960	.8418	1.1590
30	.5603	.6130	.6973	.8298	1.1755

TABLE XXVIII

Anderson-Darling
Shape Parameter = 4.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.5293	.5835	.6464	.7535	.9715
10	.5475	.5996	.6720	.8031	1.1020
15	.5435	.6006	.6816	.8154	1.1036
20	.5565	.6068	.6915	.8204	1.1021
25	.5557	.6124	.6998	.8425	1.2094
30	.5577	.6193	.6970	.8239	1.1618

APPENDIX C

Tables of the Cramer-von Mises Critical Values for the Gamma Distribution

TABLE XXIX

Cramer-von Mises
Shape Parameter = .5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.1217	.1363	.1576	.1928	.3200
10	.1327	.1530	.1795	.2259	.3569
15	.1379	.1596	.1863	.2340	.3660
20	.1425	.1625	.1918	.2526	.3743
25	.1453	.1654	.1951	.2514	.3769
30	.1440	.1649	.1986	.2568	.4170

TABLE XXX

Cramer-von Mises
Shape Parameter = 1.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.1397	.1561	.1809	.2314	.3571
10	.1341	.1511	.1775	.2210	.3573
15	.1310	.1490	.1738	.2192	.3328
20	.1316	.1484	.1748	.2220	.3327
25	.1299	.1487	.1742	.2250	.3239
30	.1319	.1501	.1757	.2228	.3415

TABLE XXXI

Cramer-von Mises
Shape Parameter = 1.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.1092	.1210	.1390	.1741	.2363
10	.1132	.1268	.1471	.1815	.2670
15	.1138	.1287	.1541	.1846	.2740
20	.1199	.1353	.1551	.1917	.2902
25	.1157	.1307	.1503	.1847	.2791
30	.1171	.1324	.1520	.1870	.2872

TABLE XXXII

Cramer-von Mises
Shape Parameter = 2.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.1007	.1118	.1261	.1481	.2096
10	.1046	.1160	.1336	.1620	.2228
15	.1040	.1154	.1325	.1627	.2409
20	.1055	.1169	.1359	.1686	.2404
25	.1051	.1174	.1356	.1695	.2481
30	.1077	.1220	.1415	.1739	.2590

TABLE XXXIII

Cramer-von Mises
Shape Parameter = 2.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.0958	.1053	.1187	.1432	.1901
10	.0969	.1083	.1263	.1522	.2194
15	.0983	.1100	.1261	.1531	.2207
20	.0991	.1097	.1256	.1531	.2207
25	.0979	.1099	.1277	.1547	.2209
30	.0979	.1091	.1255	.1543	.2237

TABLE XXXIV

Cramer-von Mises
Shape Parameter = 3.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.0927	.1011	.1145	.1367	.1898
10	.0940	.1042	.1208	.1441	.2052
15	.0954	.1075	.1234	.1501	.2085
20	.0942	.1057	.1214	.1514	.2060
25	.0935	.1050	.1224	.1524	.2137
30	.0957	.1881	.1247	.1521	.2260

TABLE XXXV

Cramer-von Mises
Shape Parameter = 3.5

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.0919	.1009	.1127	.1341	.1827
10	.0914	.1006	.1148	.1396	.2007
15	.0925	.1032	.1172	.1418	.2002
20	.0939	.1051	.1203	.1473	.2112
25	.0920	.1034	.1189	.1447	.2116
30	.0926	.1024	.1176	.1416	.2056

TABLE XXXVI

Cramer-von Mises
Shape Parameter = 4.0

Sample Size n	Level of Significance				
	.20	.15	.10	.05	.01
5	.0889	.0968	.1109	.1295	.1763
10	.0906	.1002	.1150	.1391	.1974
15	.0915	.1012	.1152	.1396	.1967
20	.0908	.1015	.1150	.1424	.1959
25	.0916	.1021	.1182	.1433	.2110
30	.0928	.1019	.1178	.1430	.2050

APPENDIX D

Graphs of the Kolmogorov-Smirnov

Critical Values Verses the

Gamma Shape Parameters

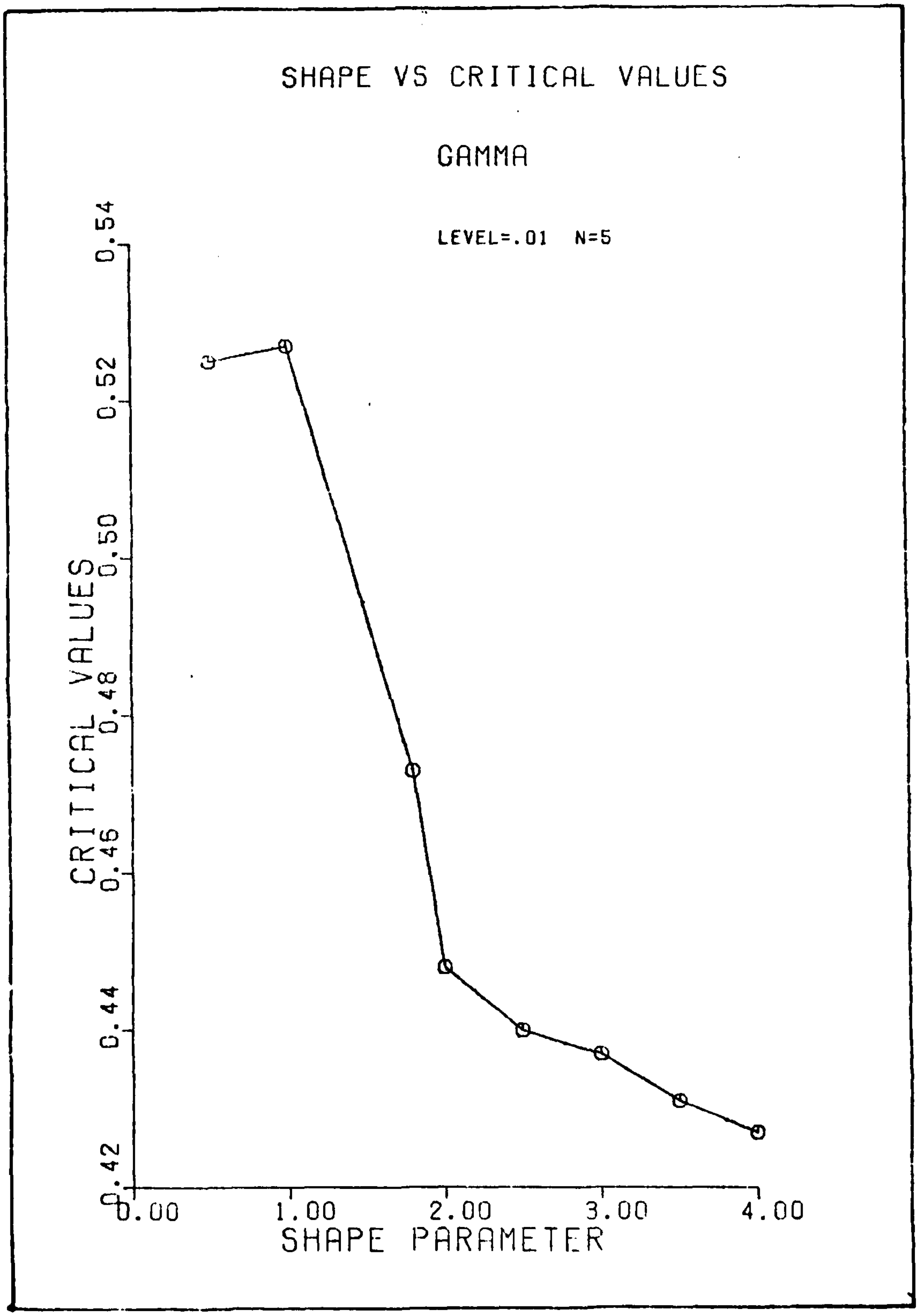


FIG 7. Shape vs K-S Critical Values, Level = .01, n = 5

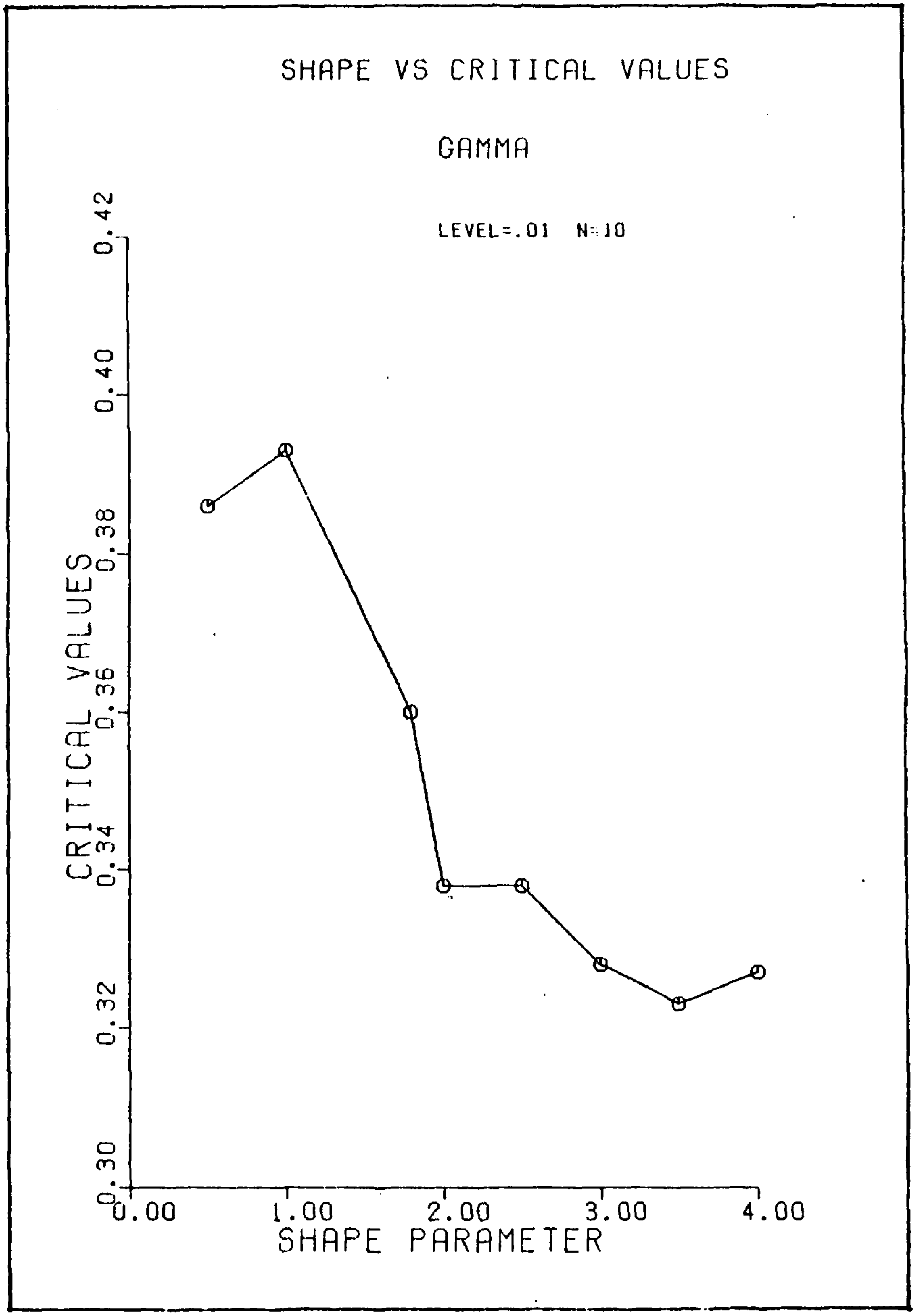


FIG 8. Shape vs K-S Critical Values, Level = .01, n = 10

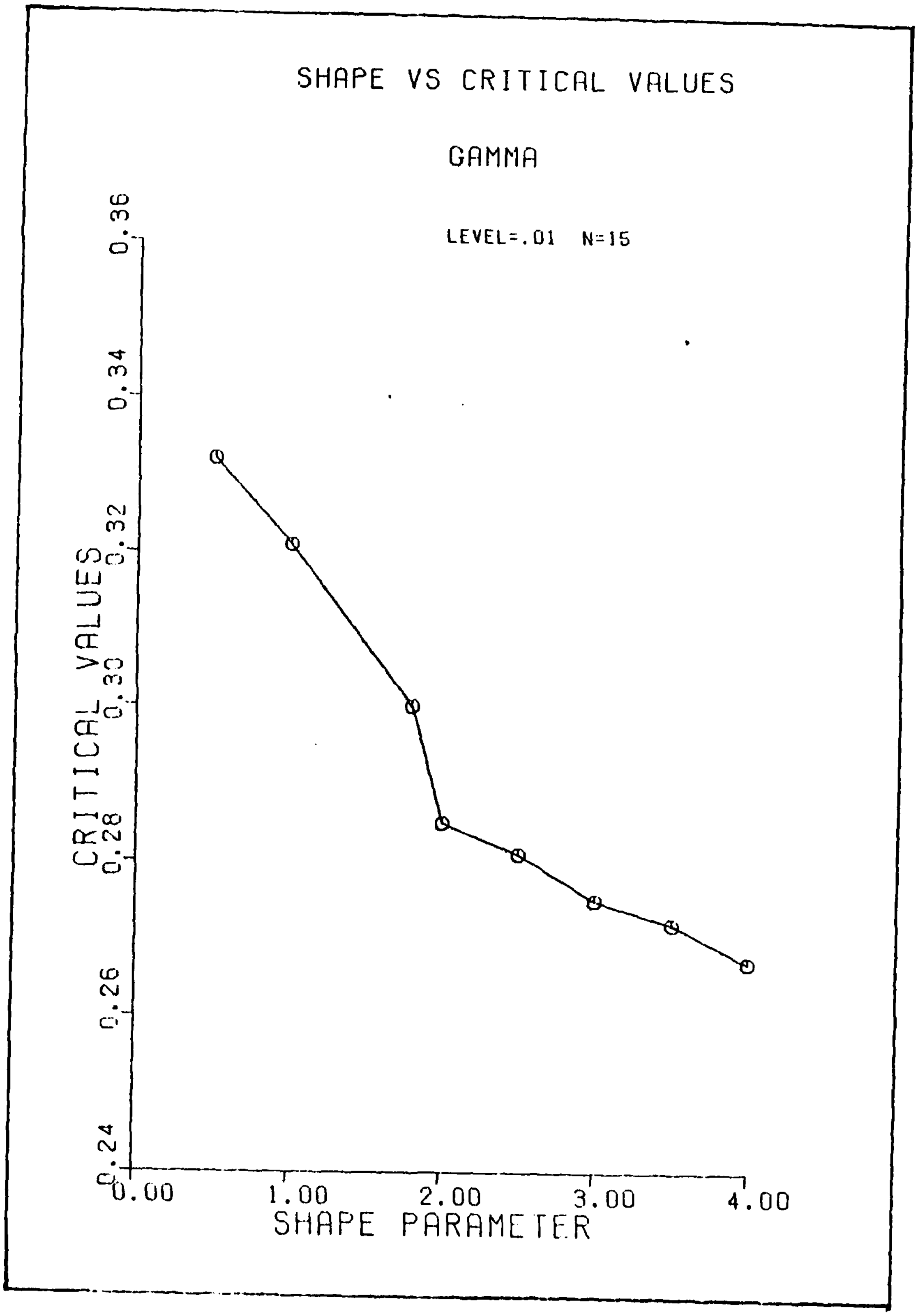


FIG 9. Shape vs K-S Critical Values, Level = .01, n = 15

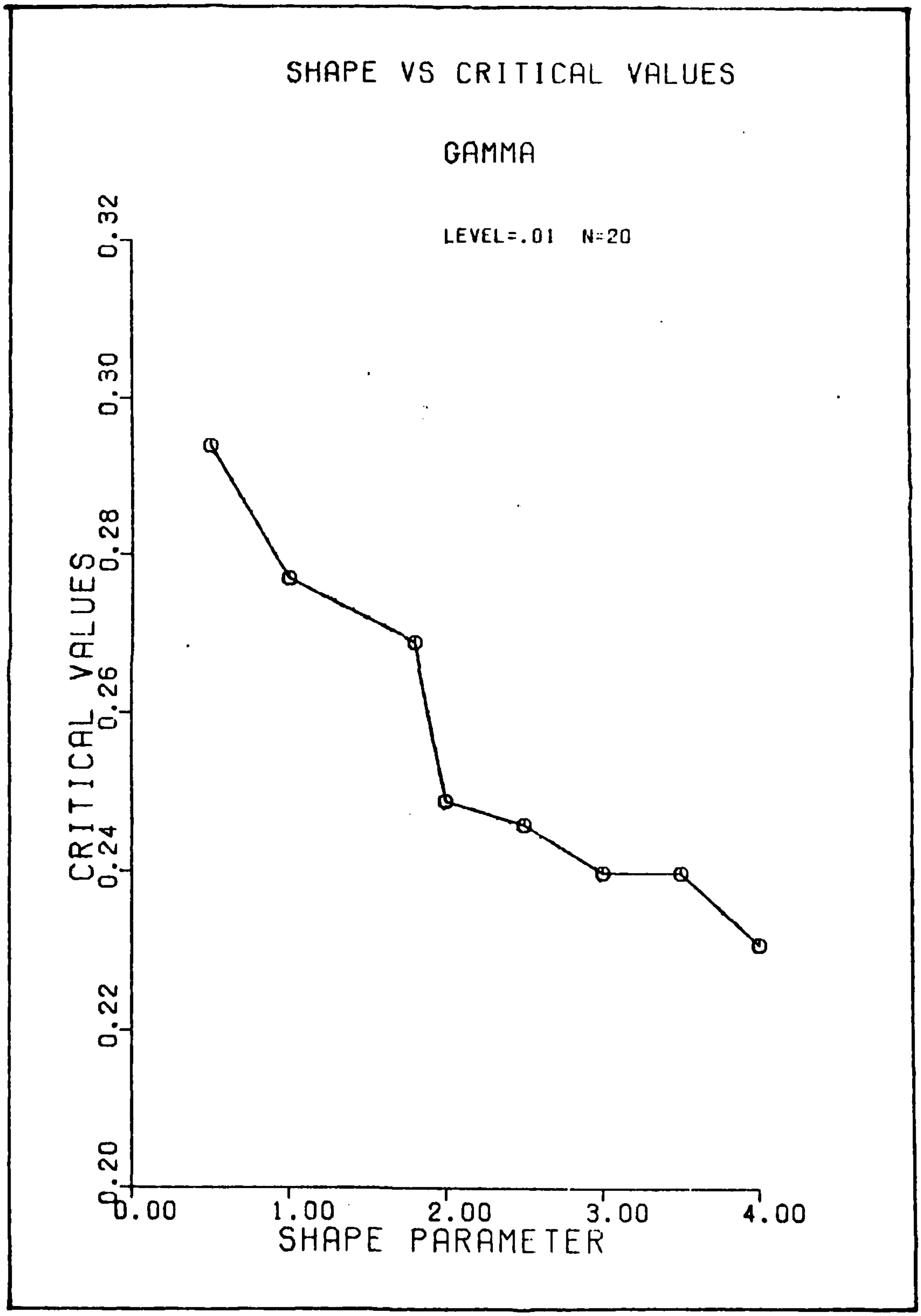


FIG 16. Shape vs K-S Critical Values, Level = .01, n = 20

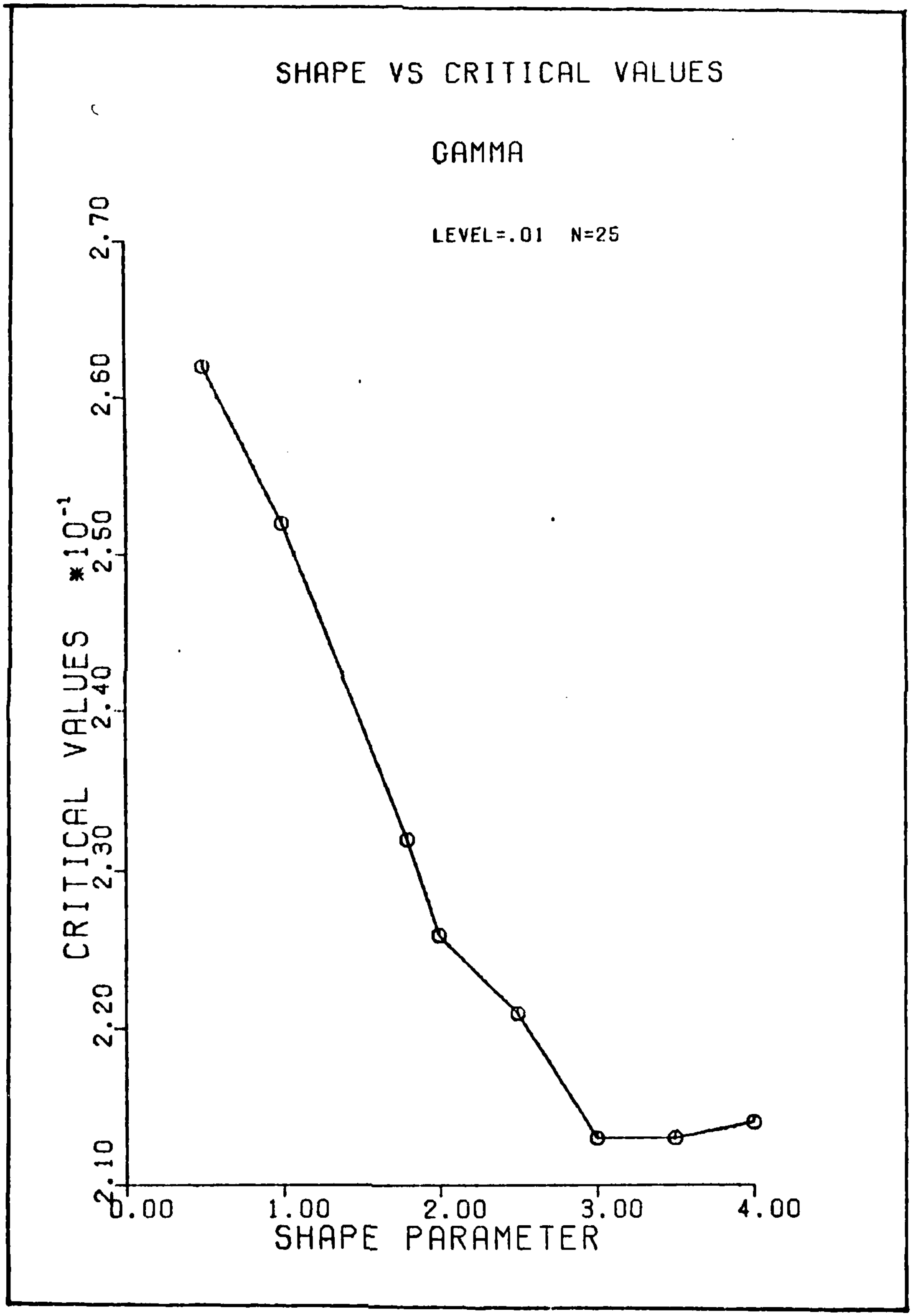


FIG 11. Shape vs K-S Critical Values, Level = .01, n = 25

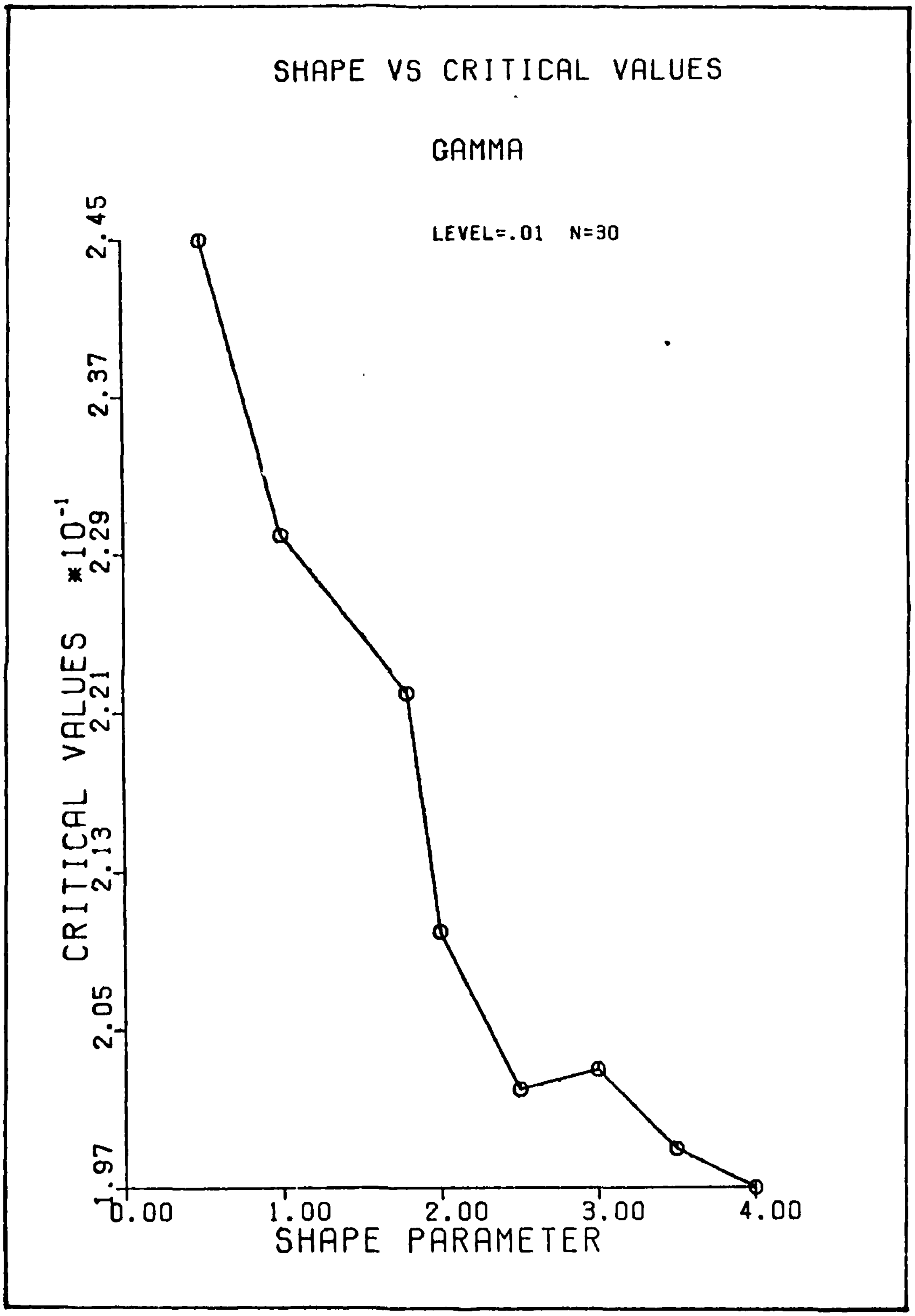


FIG 12. Shape vs K-S Critical Values, Level = .01, n = 30

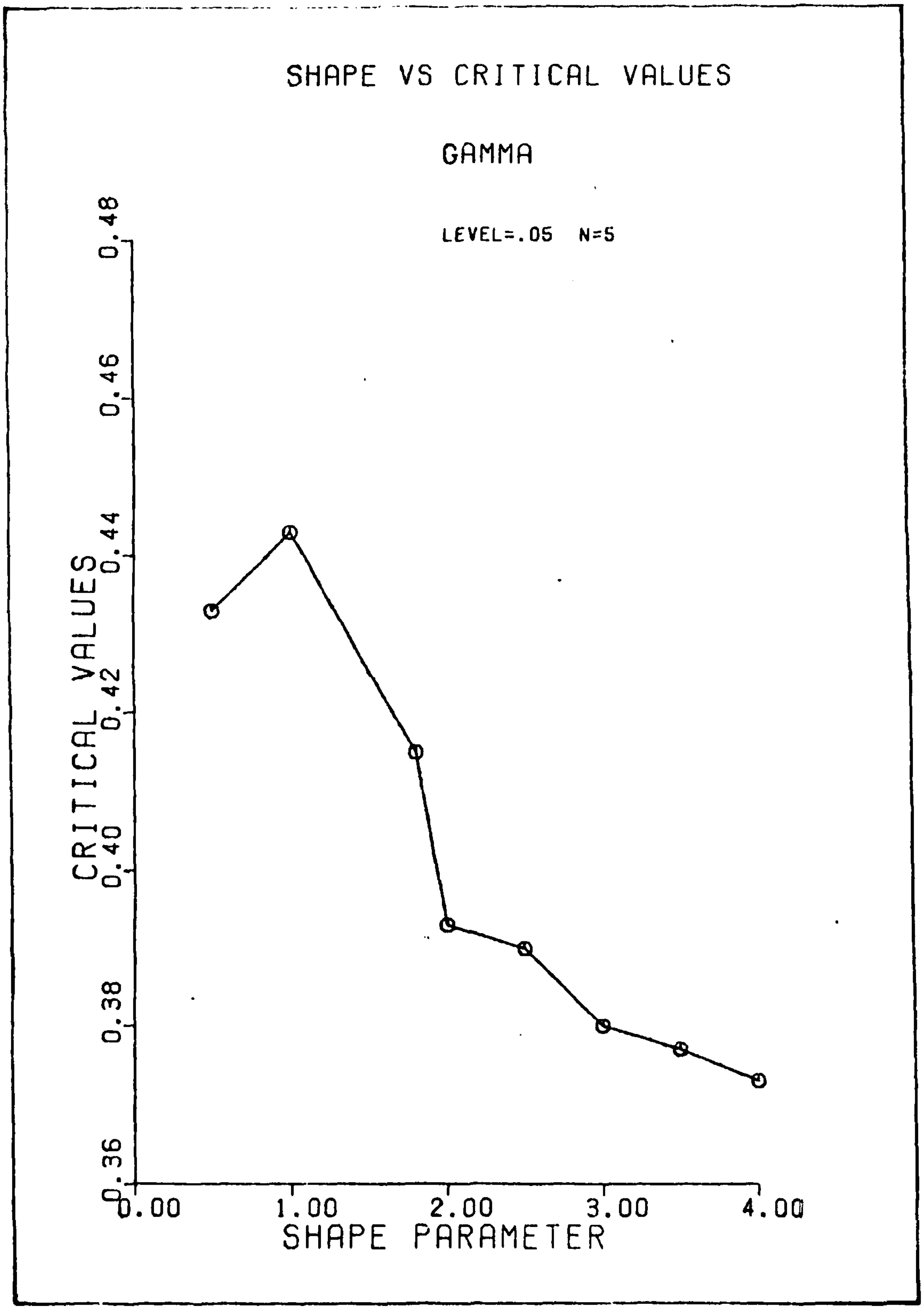


FIG 13. Shape vs K-S Critical Values, Level = .05, n = 5

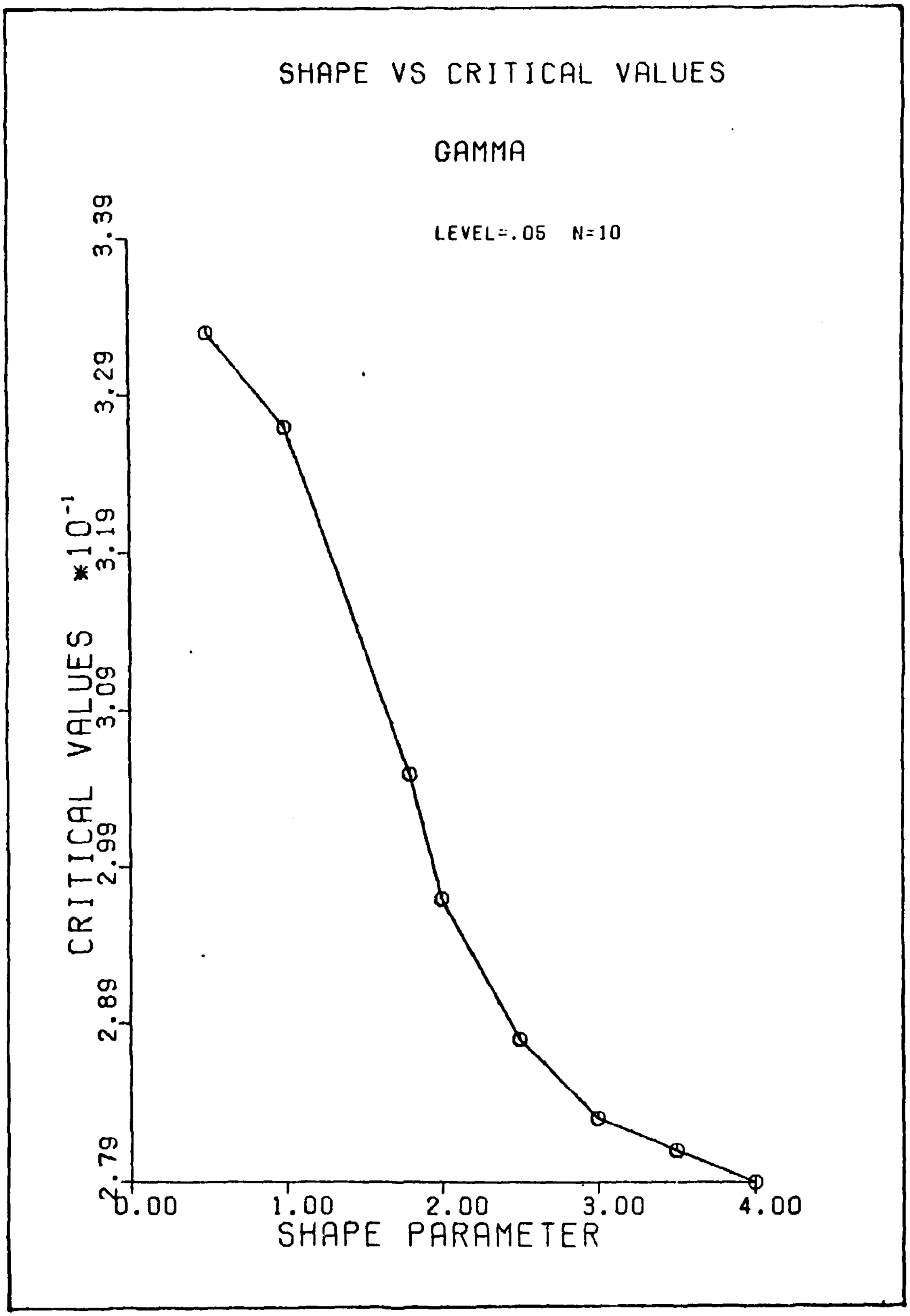


FIG 14. Shape vs K-S Critical Values, Level = .05, n = 10

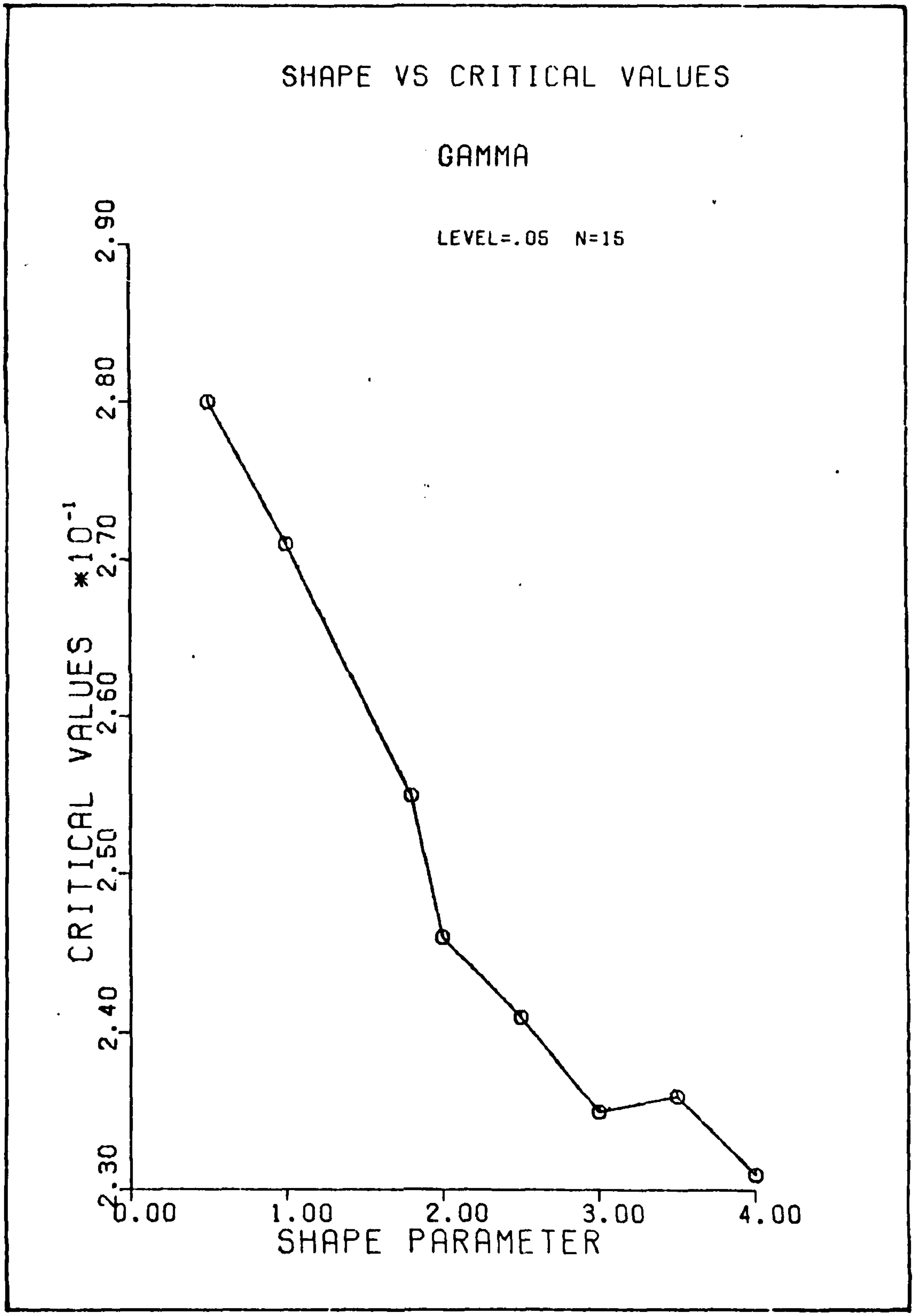


FIG 15. Shape vs K-S Critical Values, Level = .05, n = 15

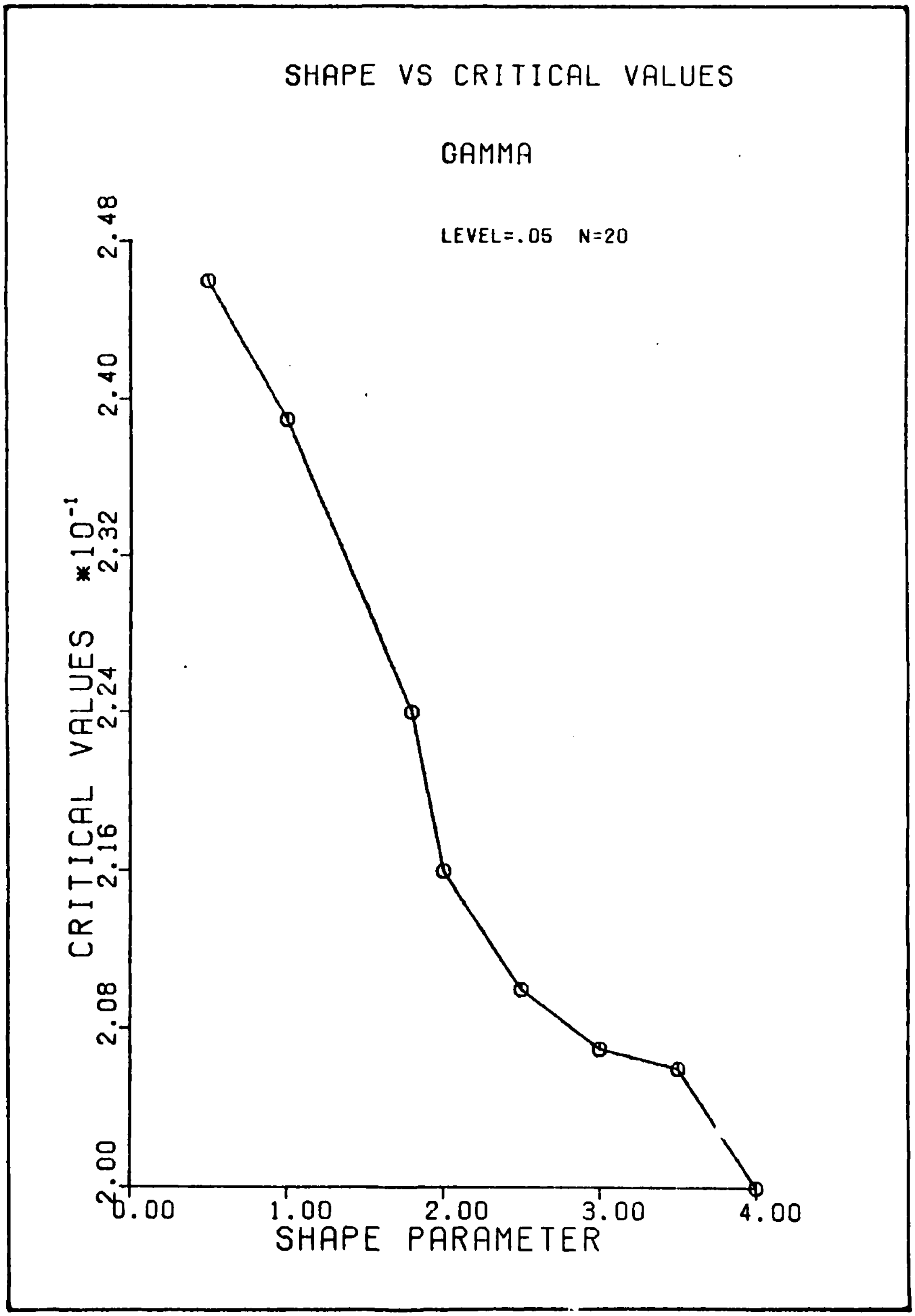


FIG 16. Shape vs K-S Critical Values, Level = .05, n = 20

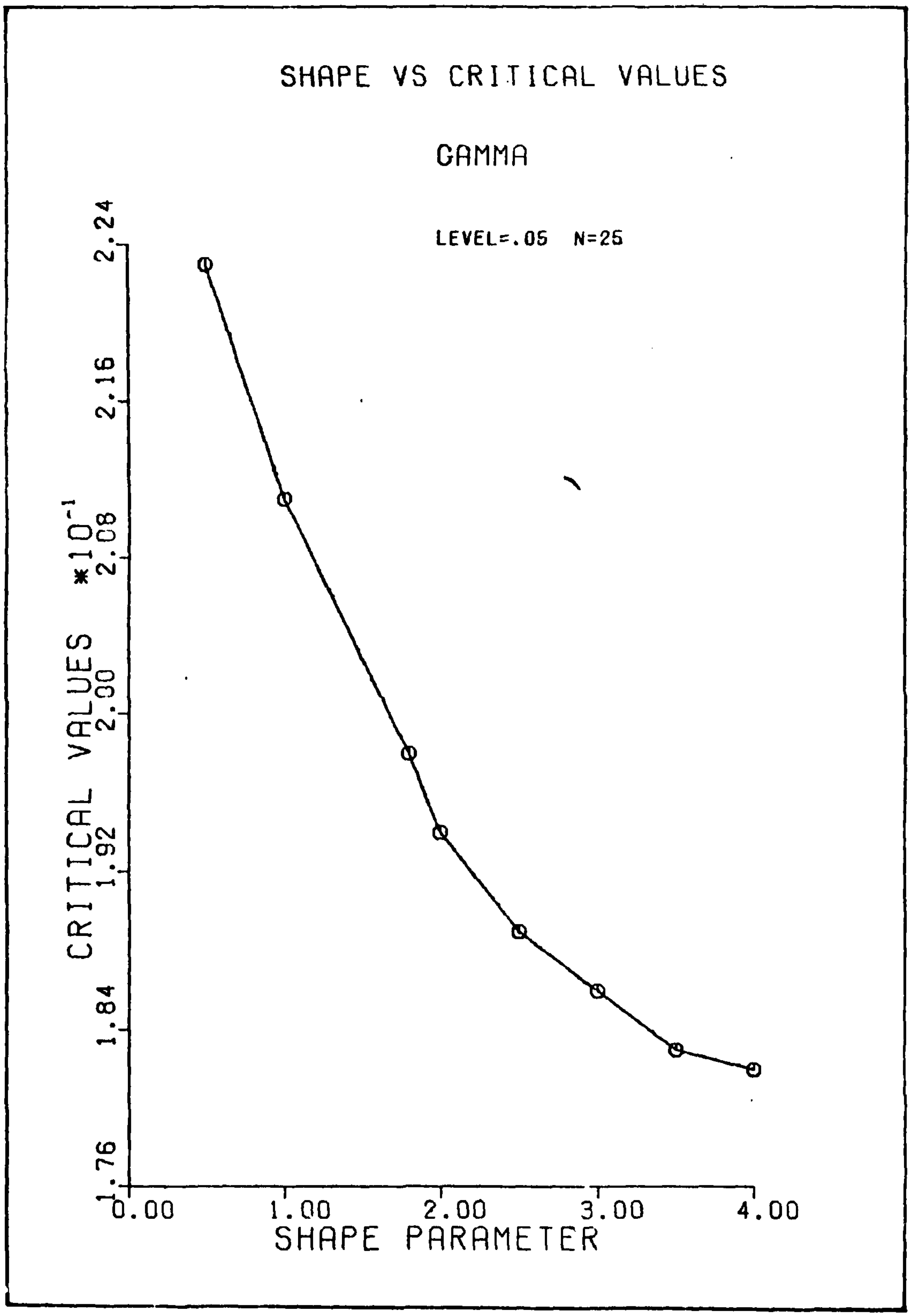


FIG 17. Shape vs K-S Critical Values, Level = .05, n = 25

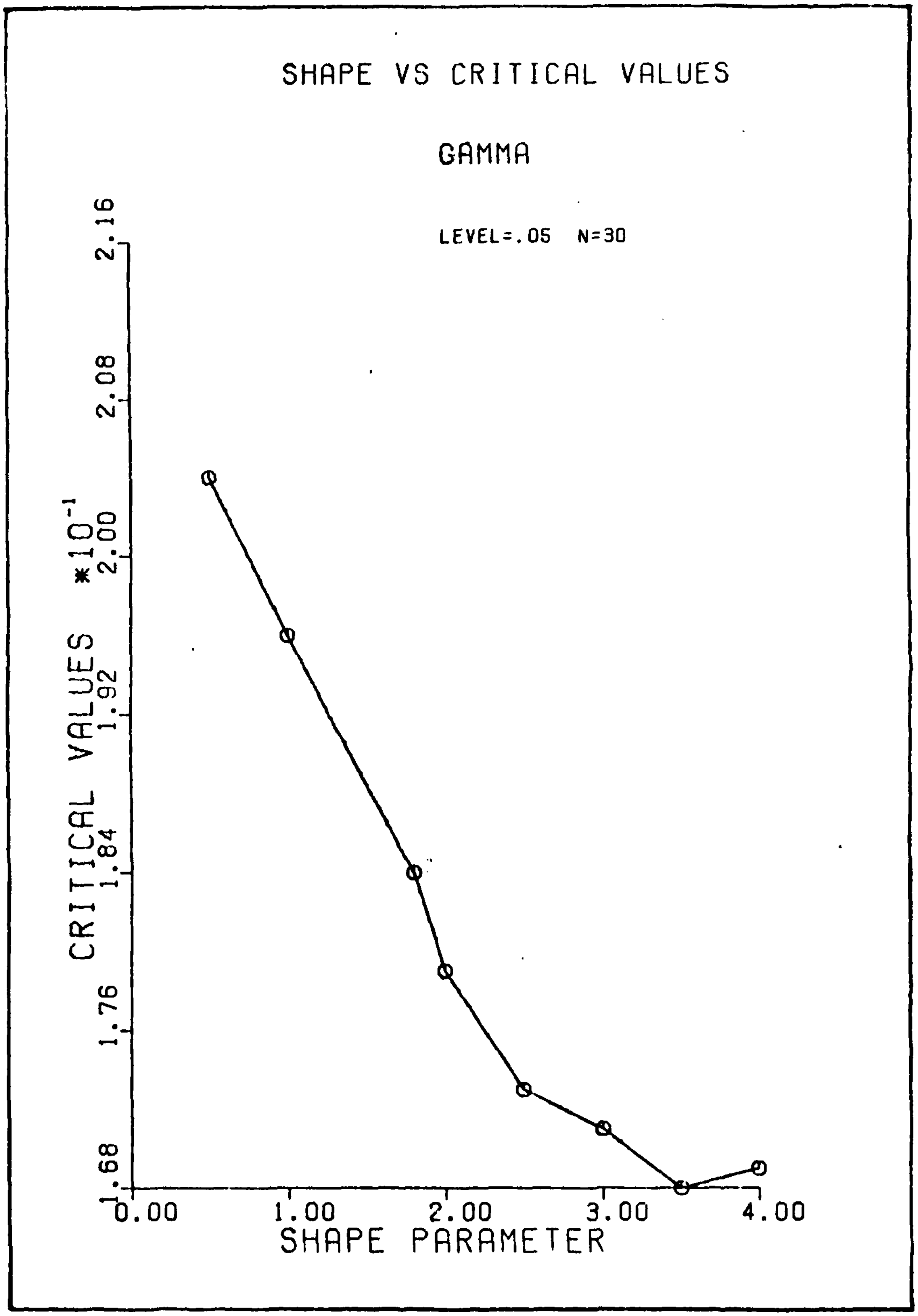


FIG 18. Shape vs K-S Critical Values, Level = .05, n = 30

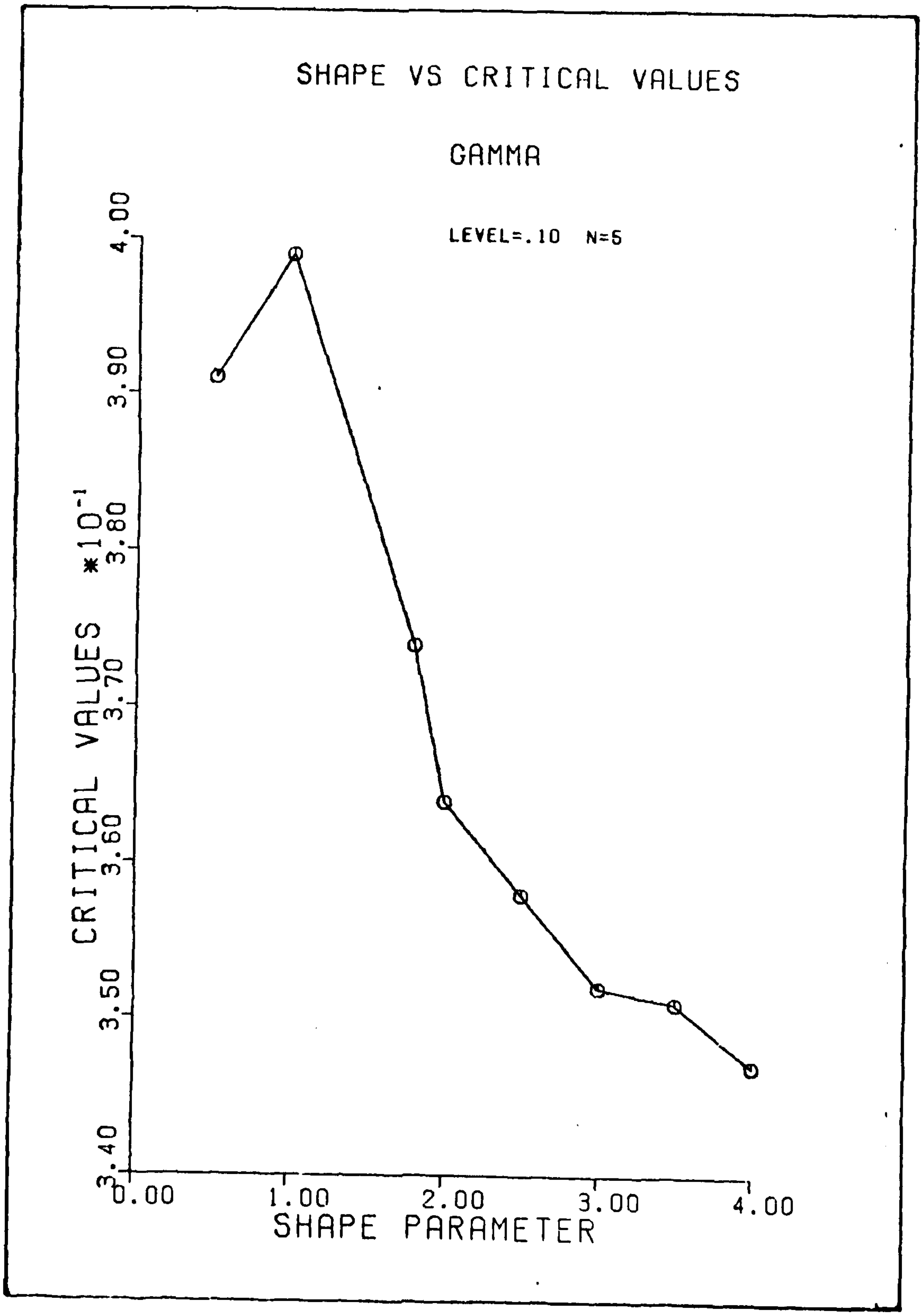


FIG. 19. Shape vs K-S Critical Values, Level = .10, n = 5

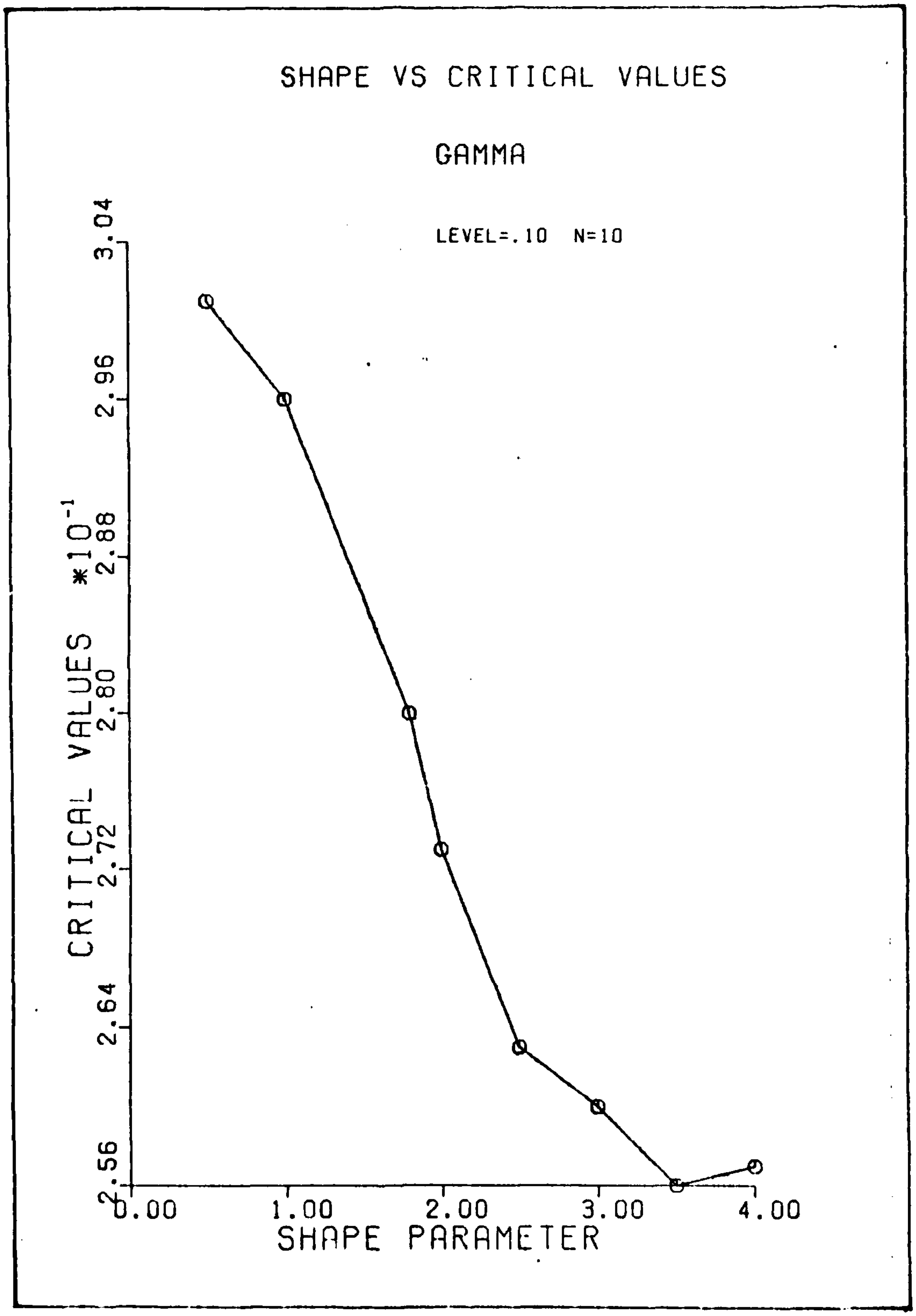
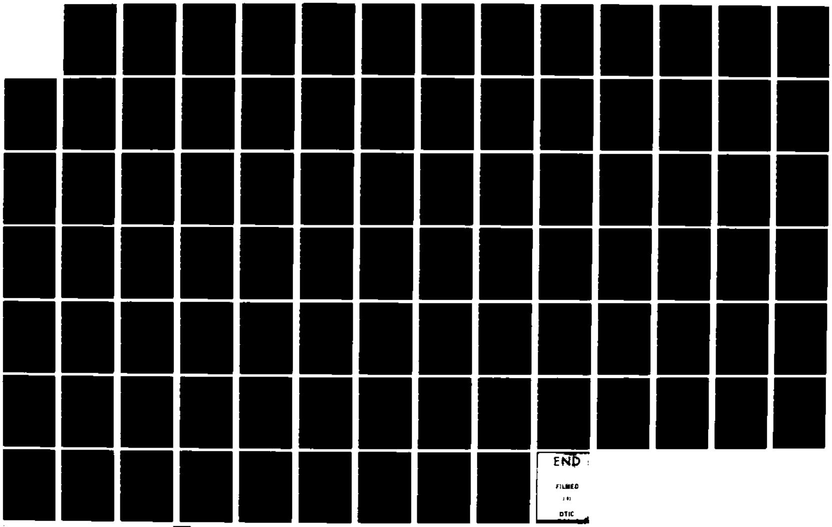
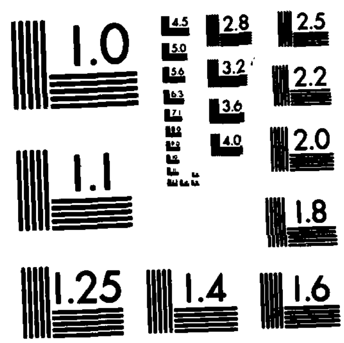


FIG 20. Shape vs K-S Critical Values, Level = .10, n = 10

AD-A124 841 A MODIFIED KOLMOGOROV-SMIRNOV ANDERSON-DARLING AND CRAMER-VON MISES TEST F..(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. P J VIVIANO 2/2

UNCLASSIFIED DEC 82 AFIT/GOR/MA/82D-4 F/G 12/1 NL





MICROCOPY RESOLUTION TEST CHART
NATIONAL BUREAU OF STANDARDS-1963-A

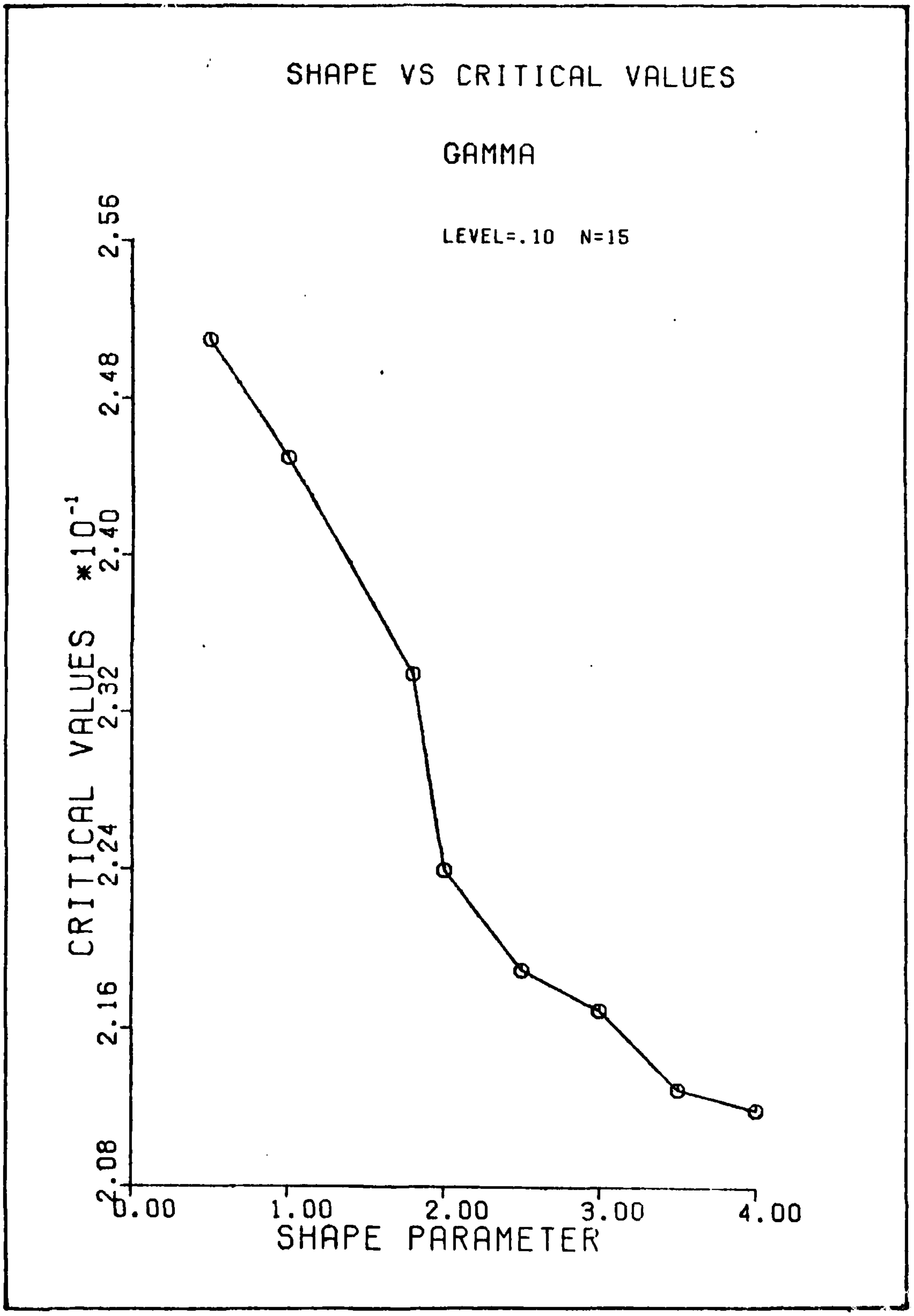


FIG 21. Shape vs K-S Critical Values, Level = .10, n = 15

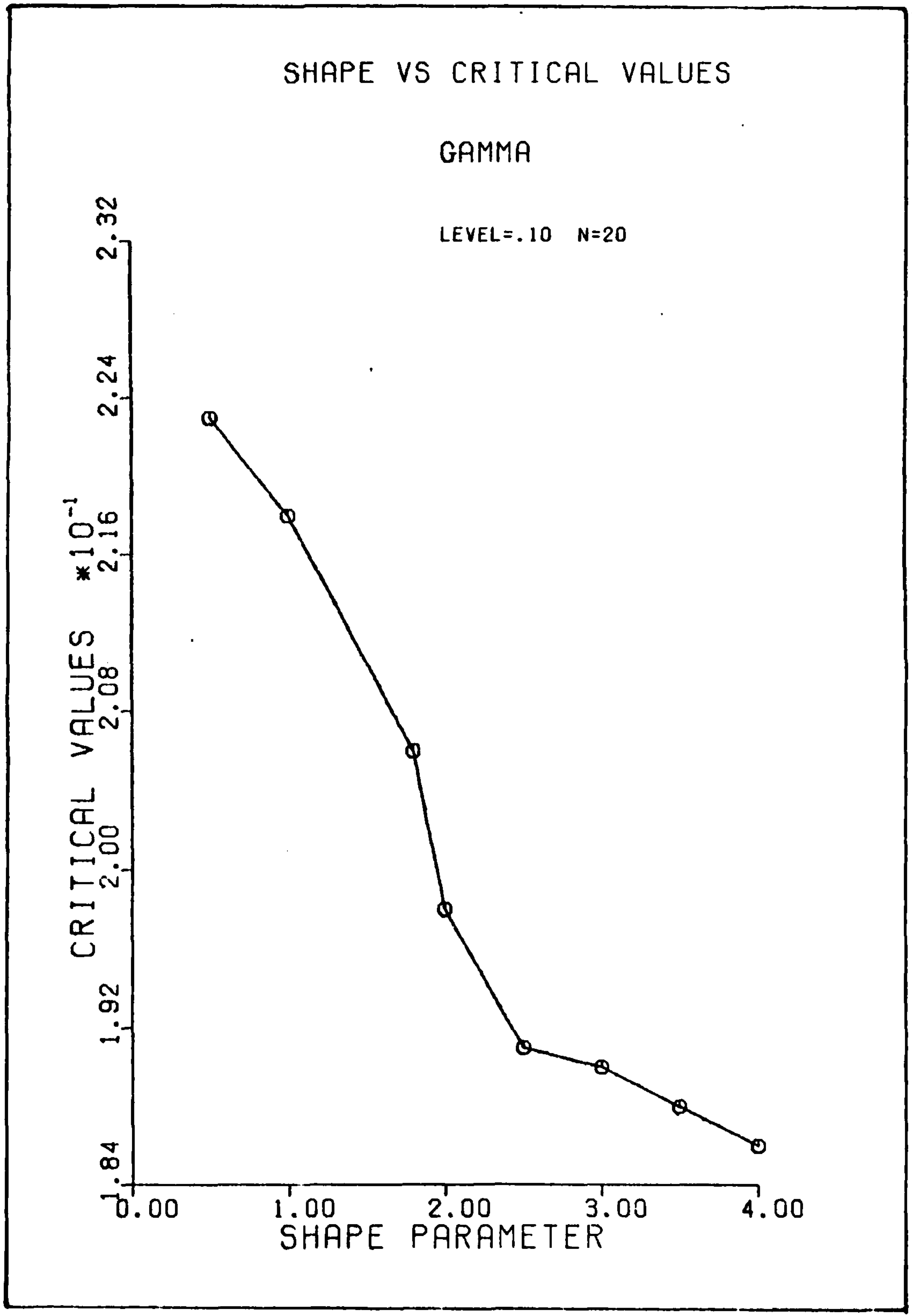


FIG 22. Shape vs K-S Critical Values, Level = .10, n = 20

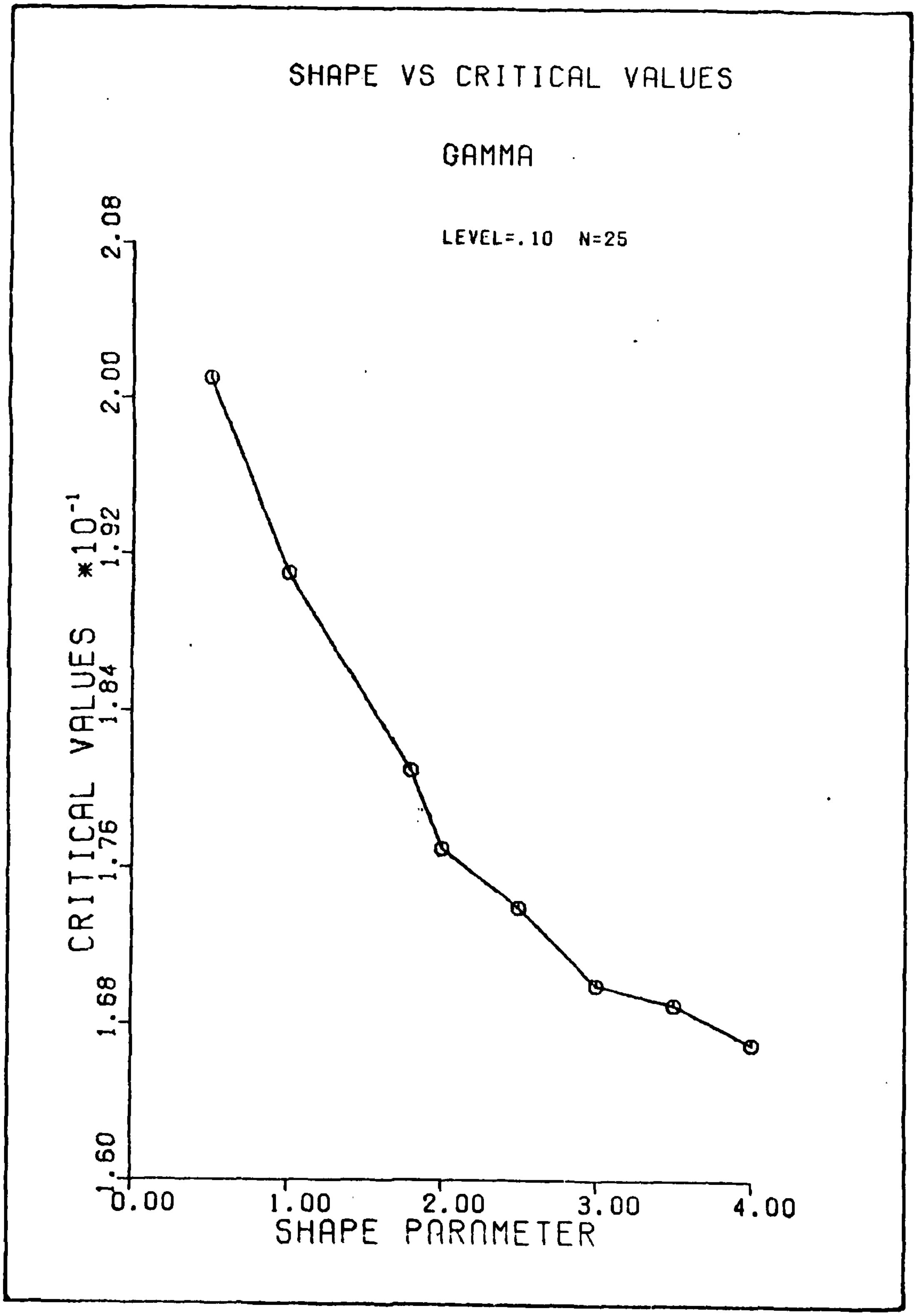


FIG 23. Shape vs K-S Critical Values, Level = .10, n = 25

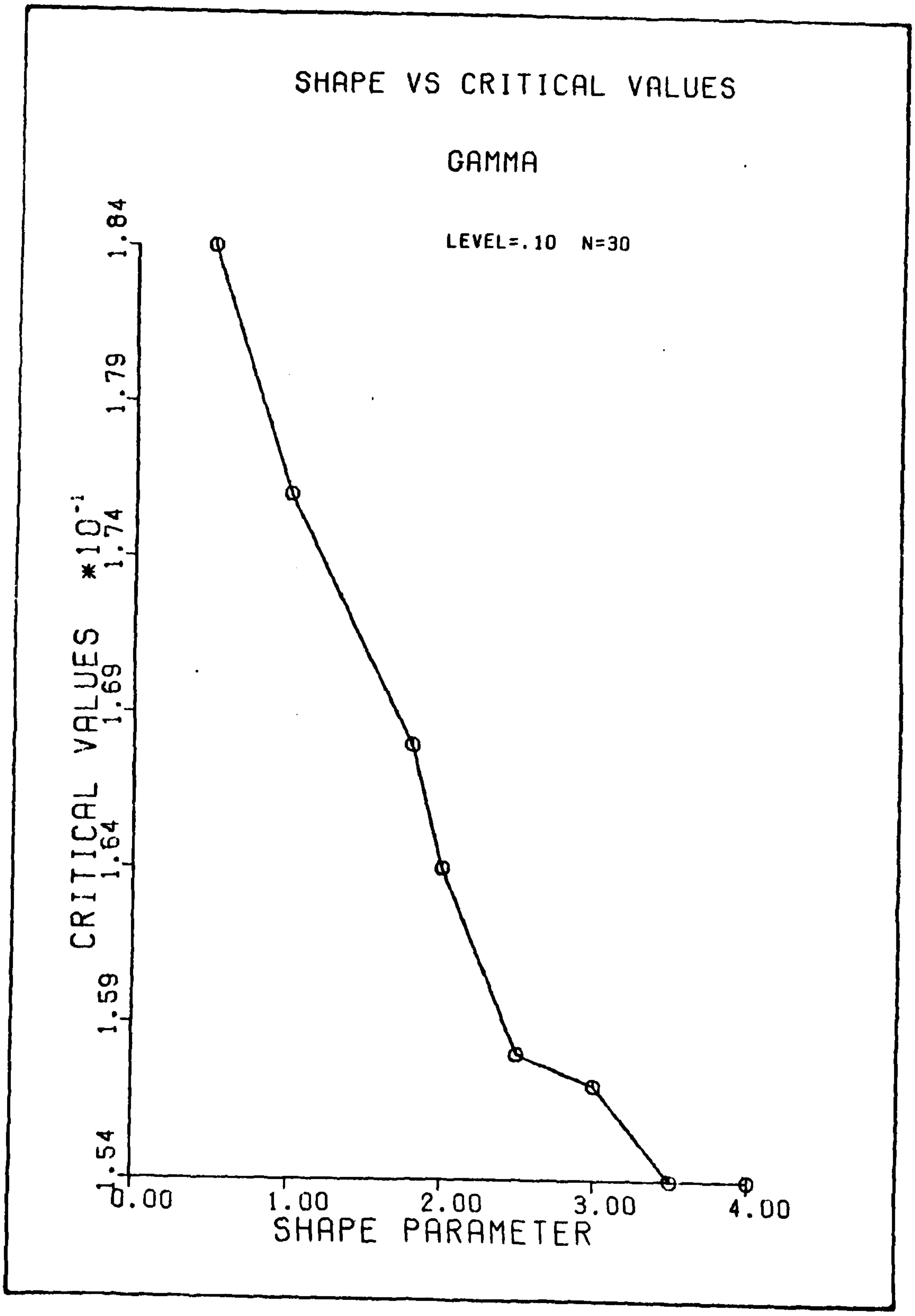


FIG 24. Shape vs K-S Critical Values, Level = .10, n = 30

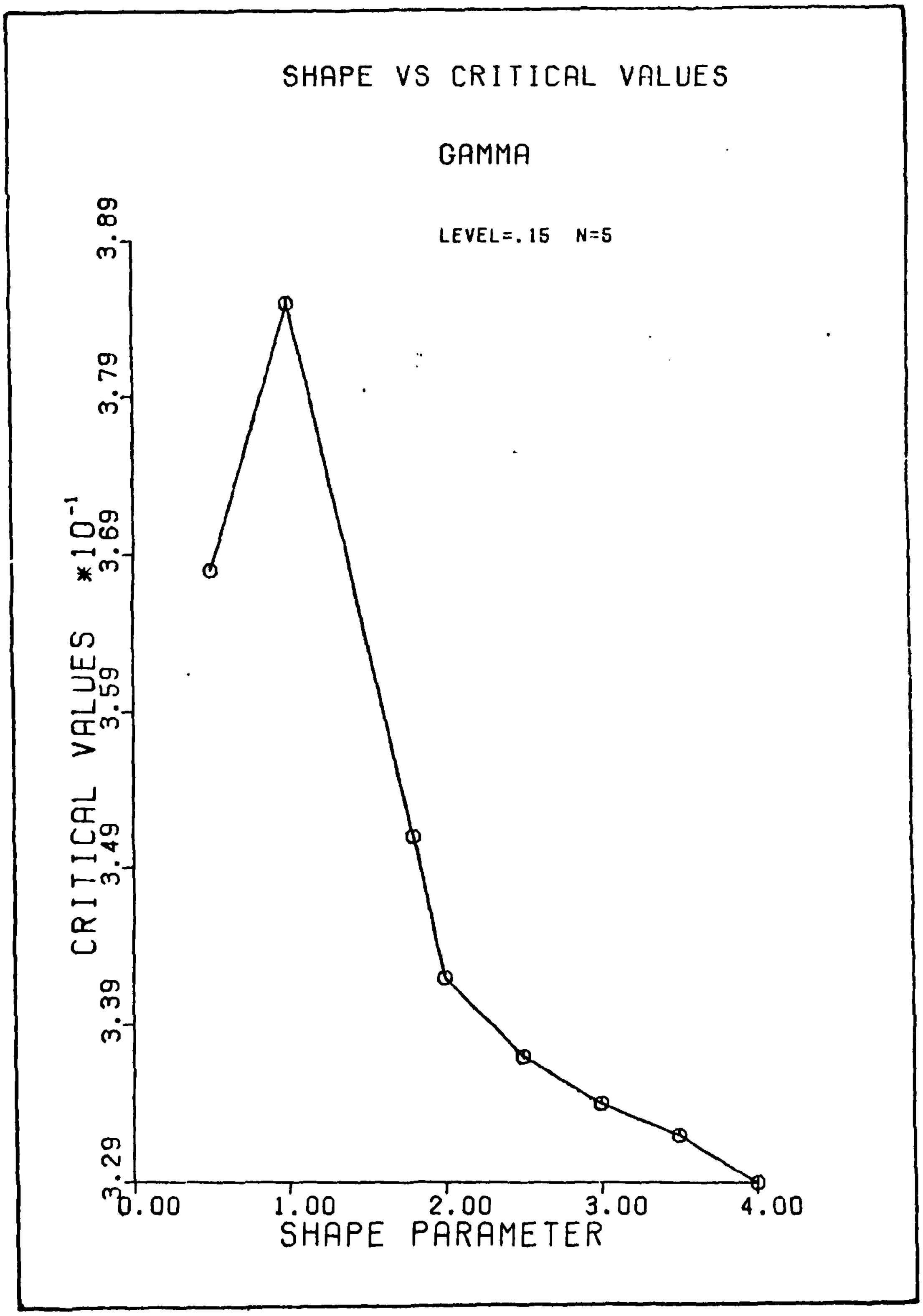


FIG 25. Shape vs K-S Critical Values, Level = .15, n = 5

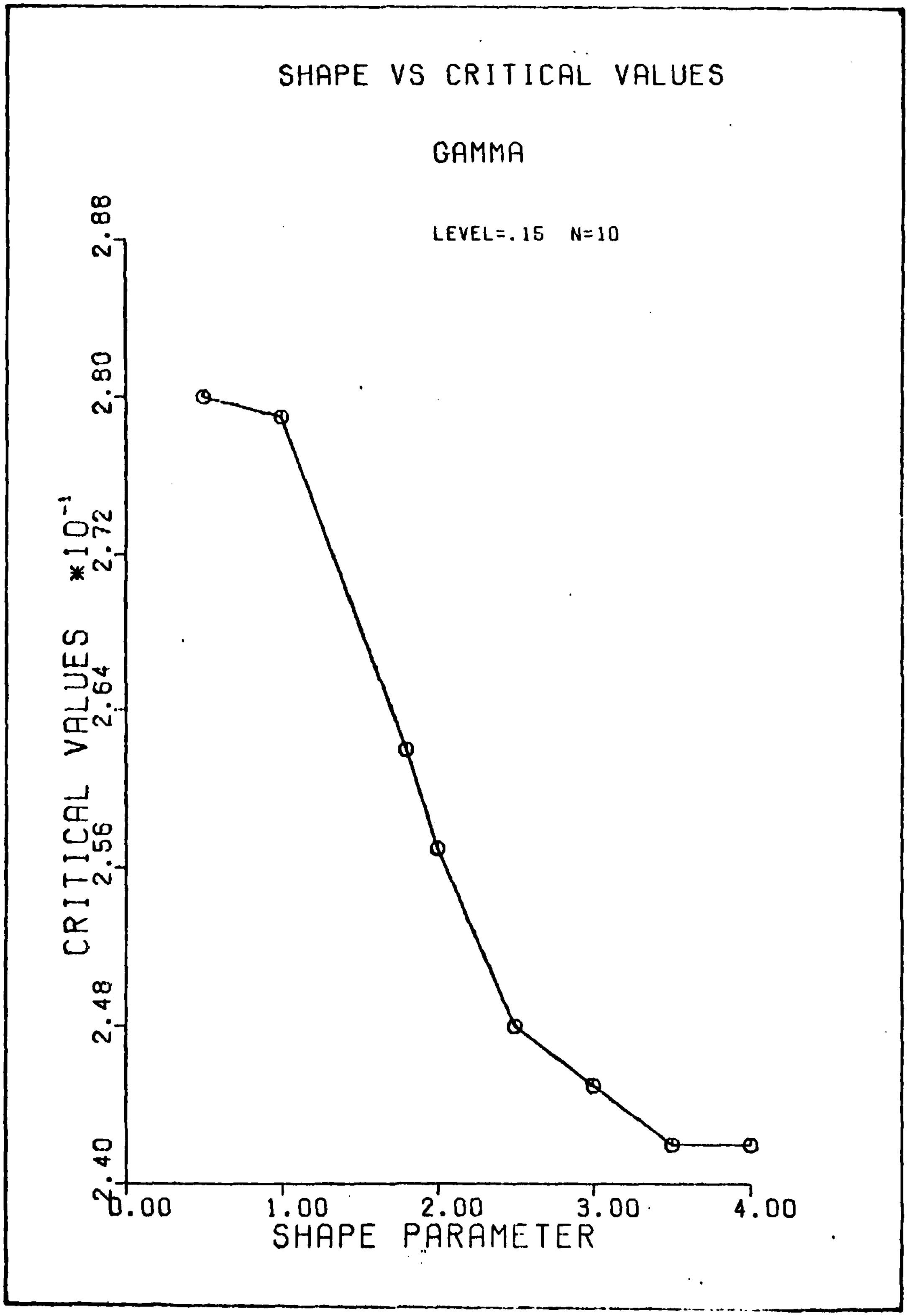


FIG 26. Shape vs K-S Critical Values, Level = .15, n = 10

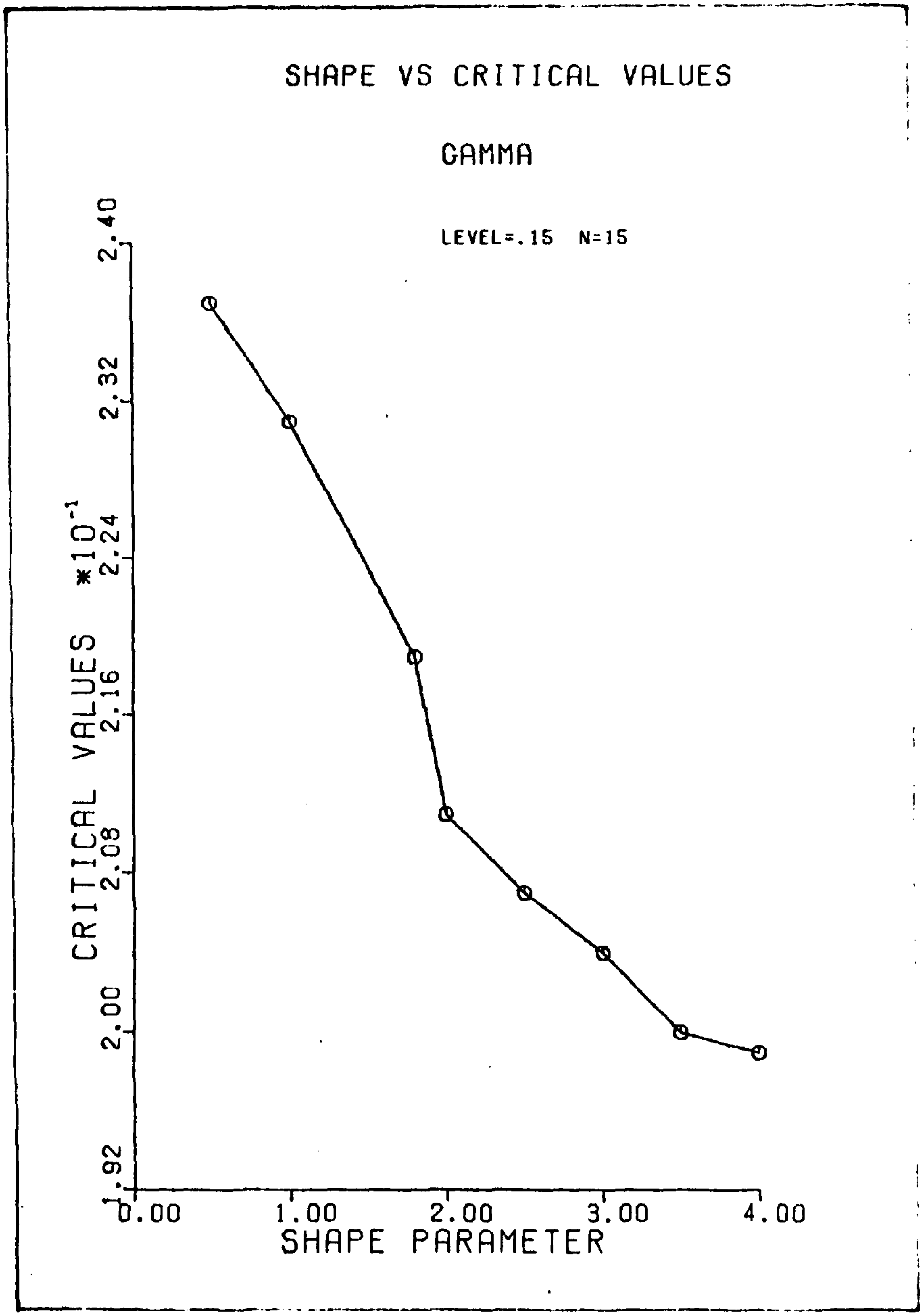


FIG 27. Shape vs K-S Critical Values, Level = .15, n = 15

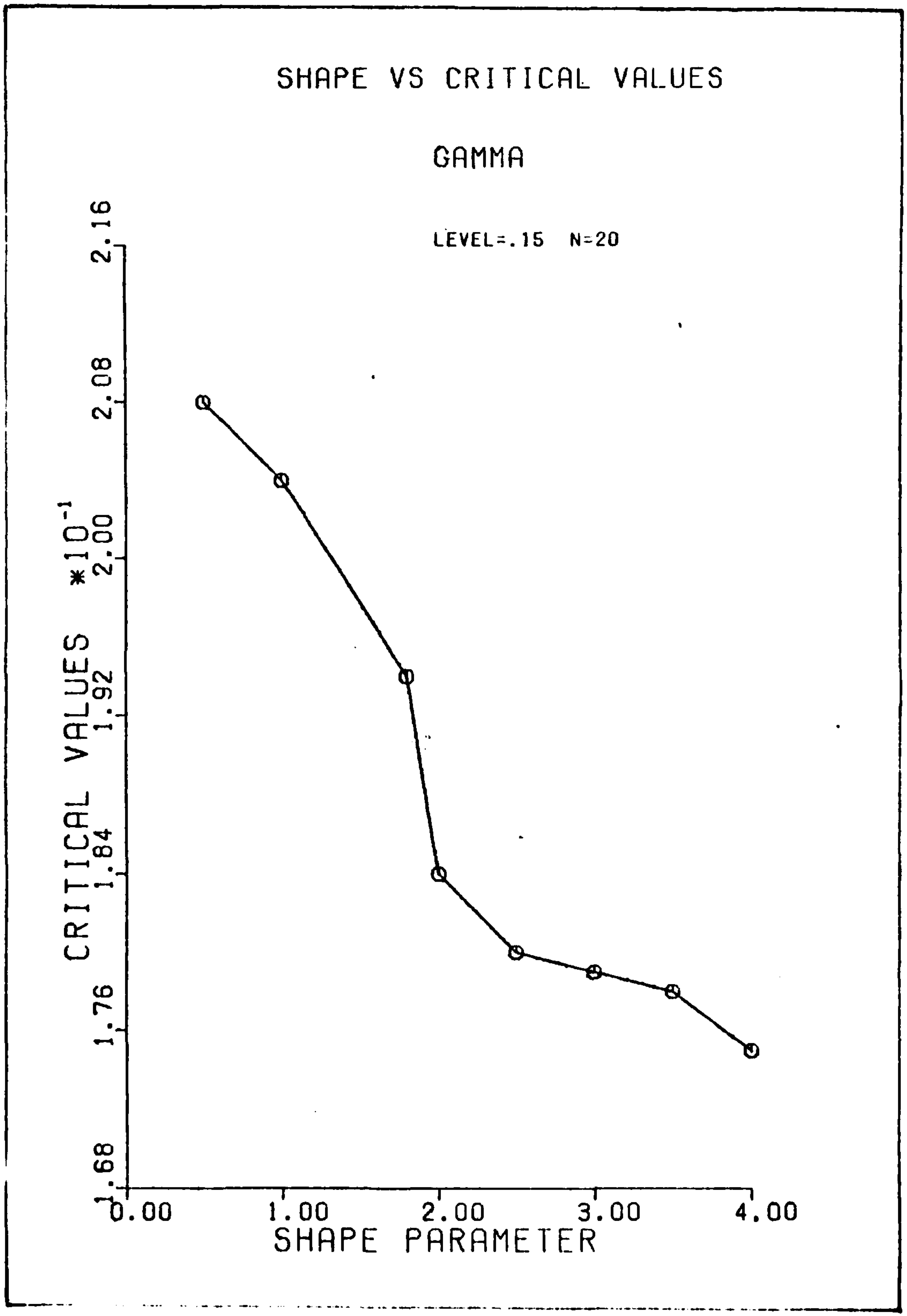


FIG 28. Shape vs K-S Critical Values, Level = .15, n = 20

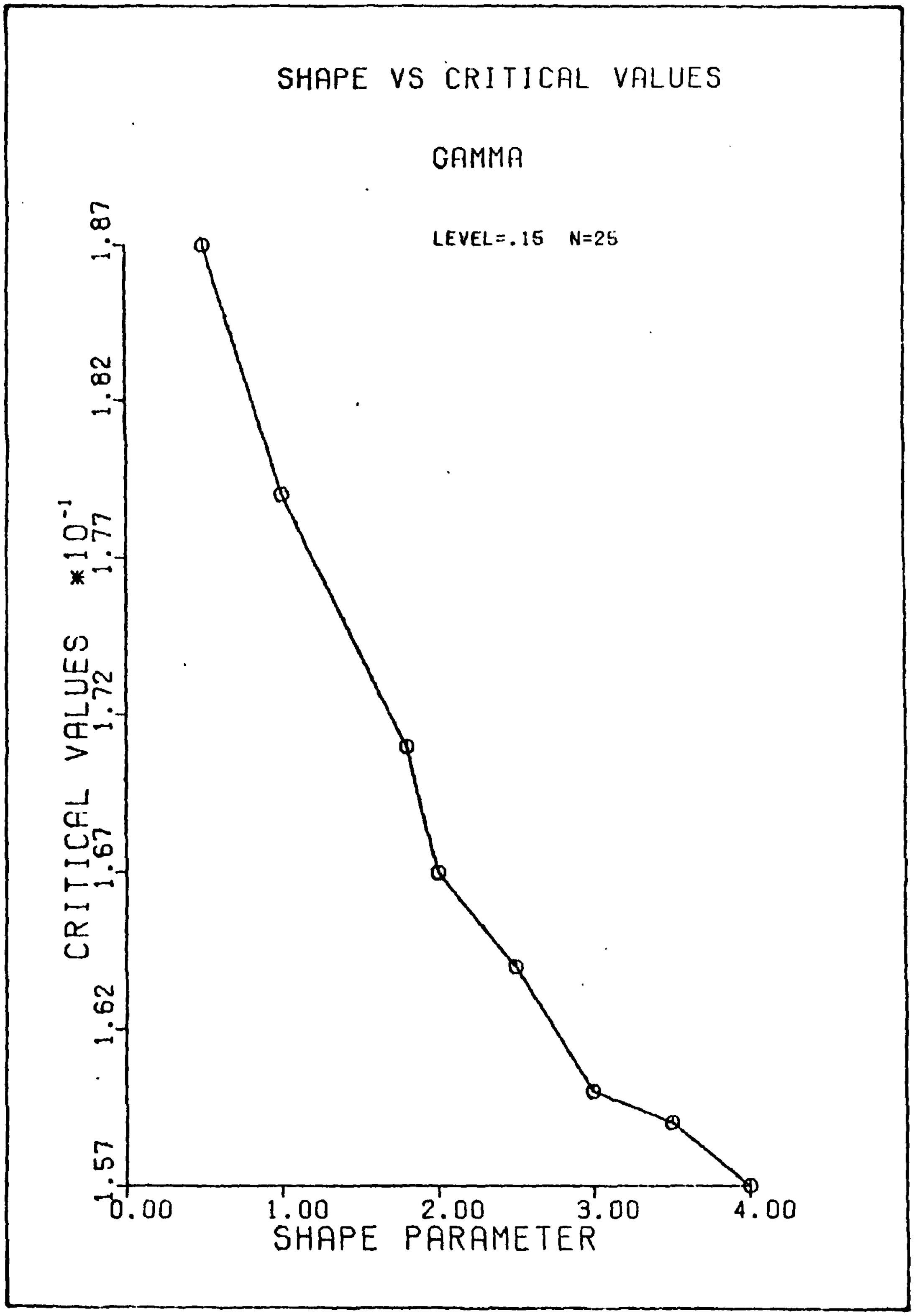


FIG 29. Shape vs K-S Critical Values, Level = .15, n = 25

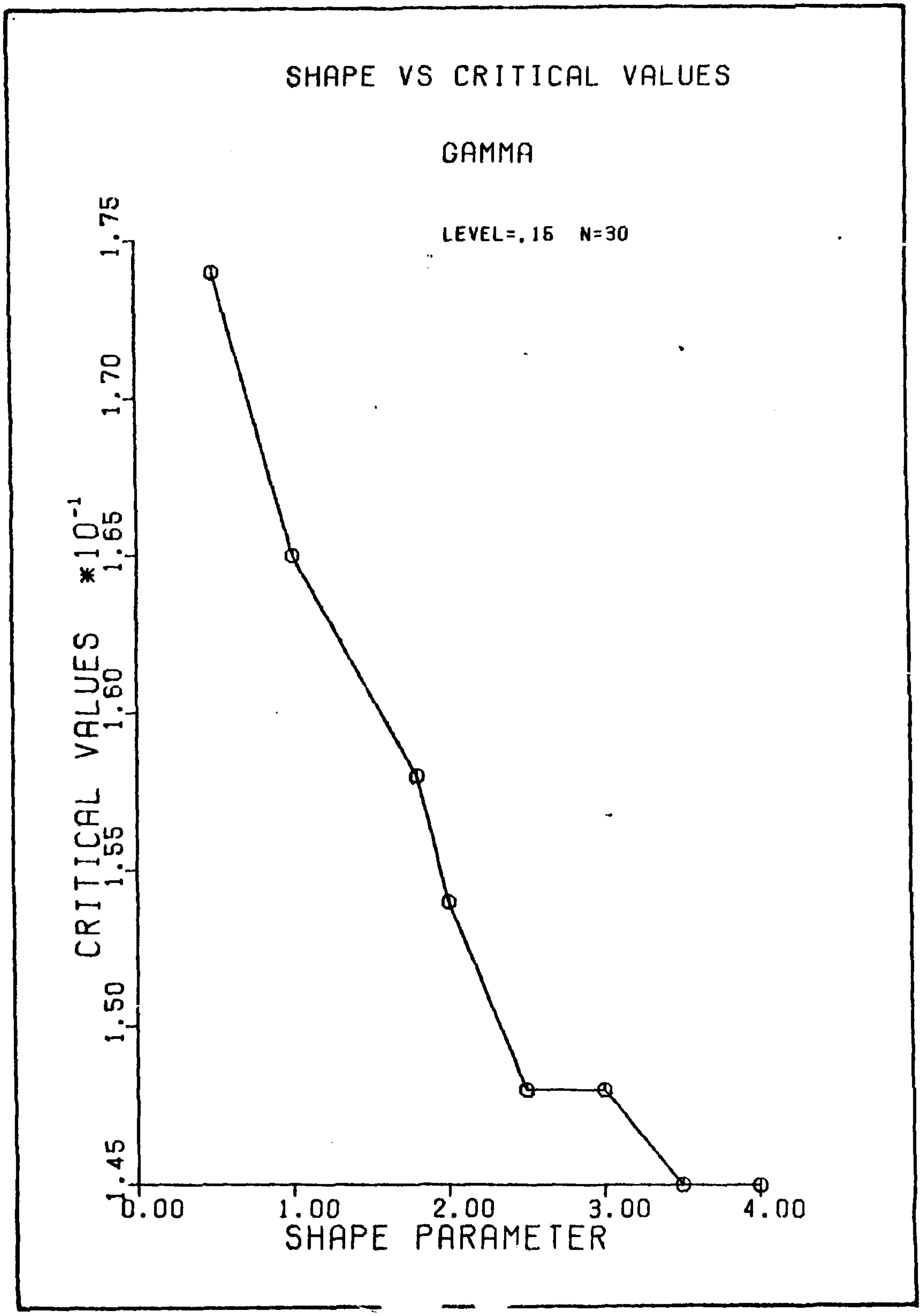


FIG 30. Shape vs K-S Critical Values, Level = .15, n = 30

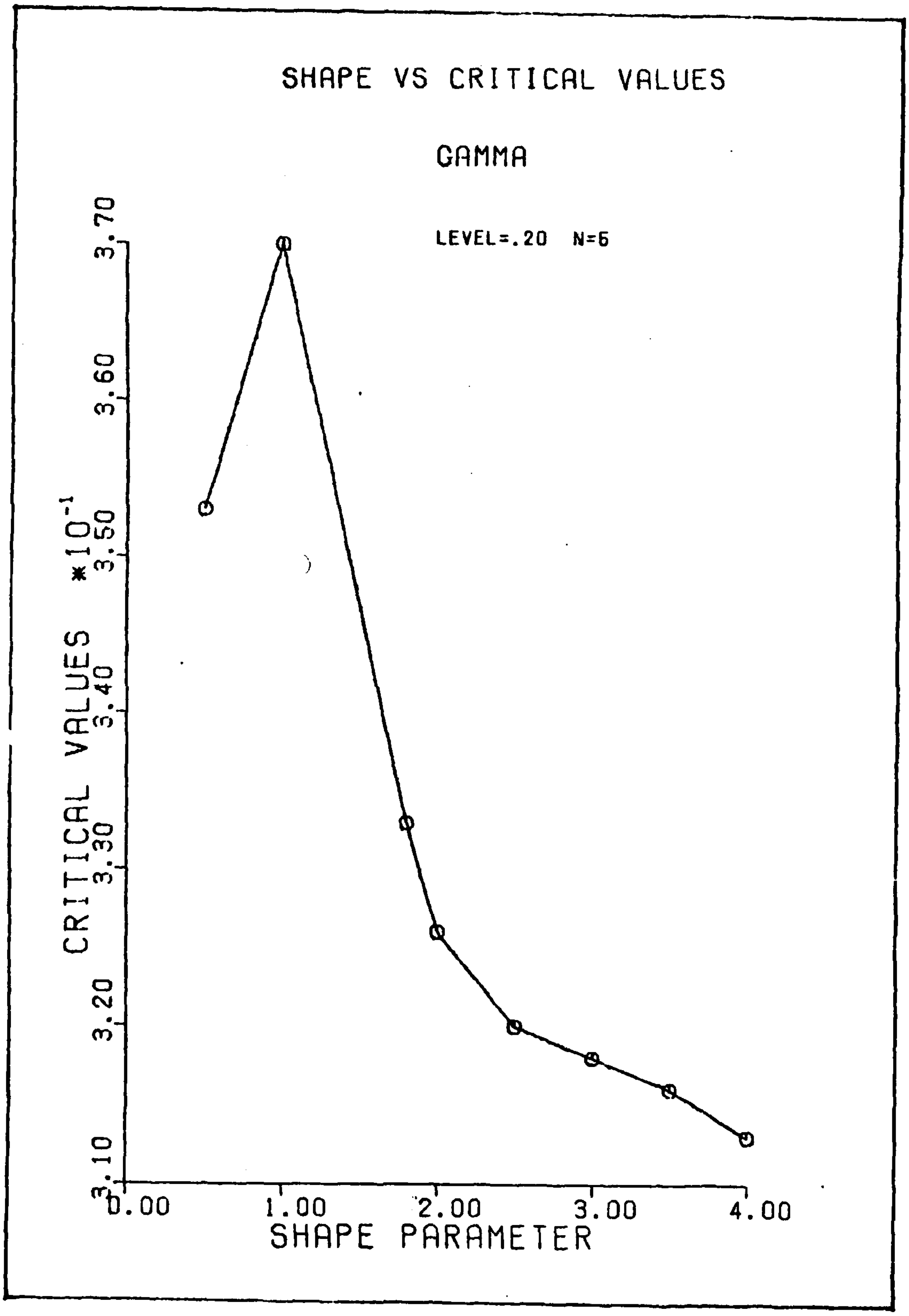


FIG 31. Shape vs K-S Critical Values, Level = .20, n = 5

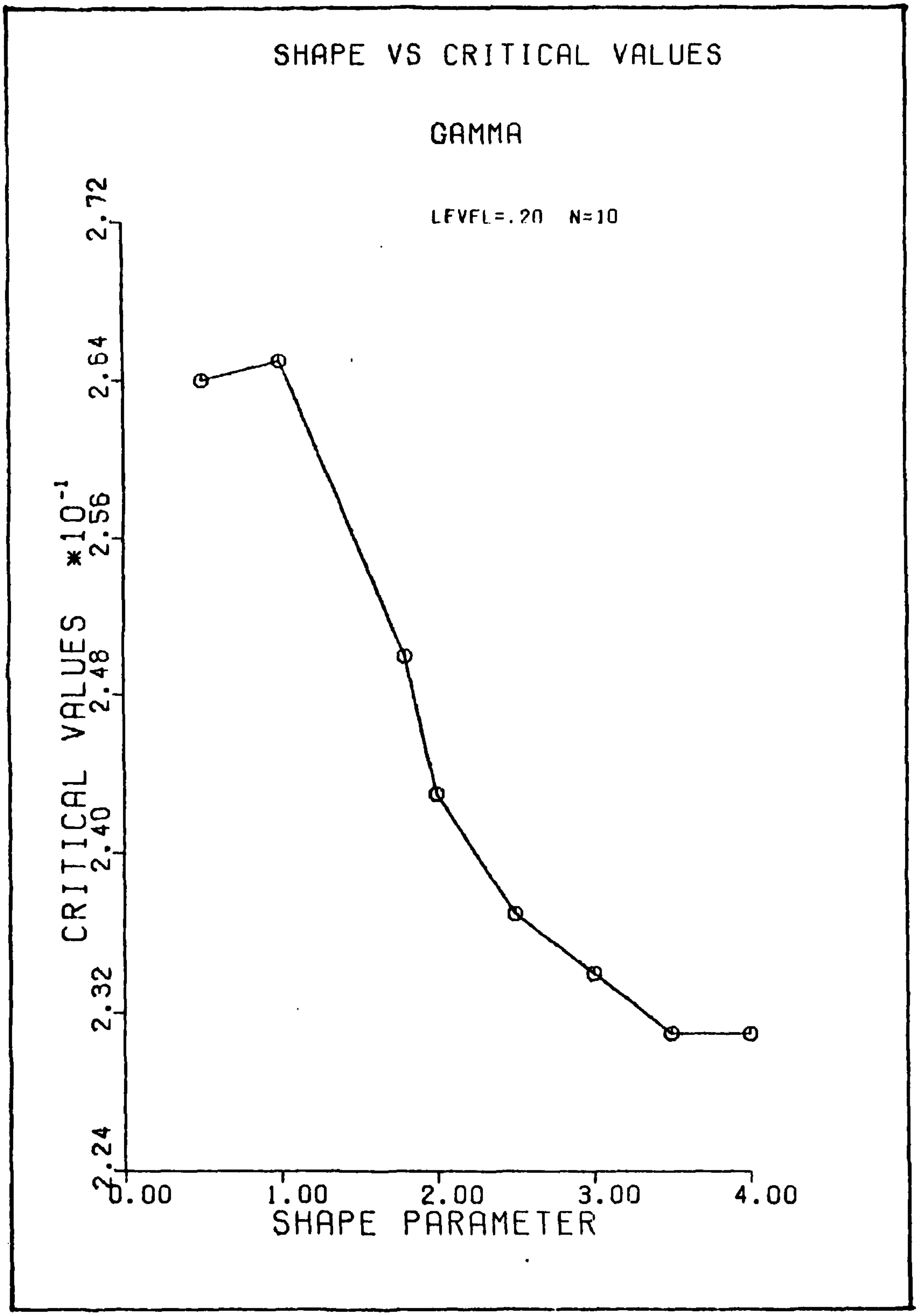


FIG 32. Shape vs K-S Critical Values, Level = .20, n = 10

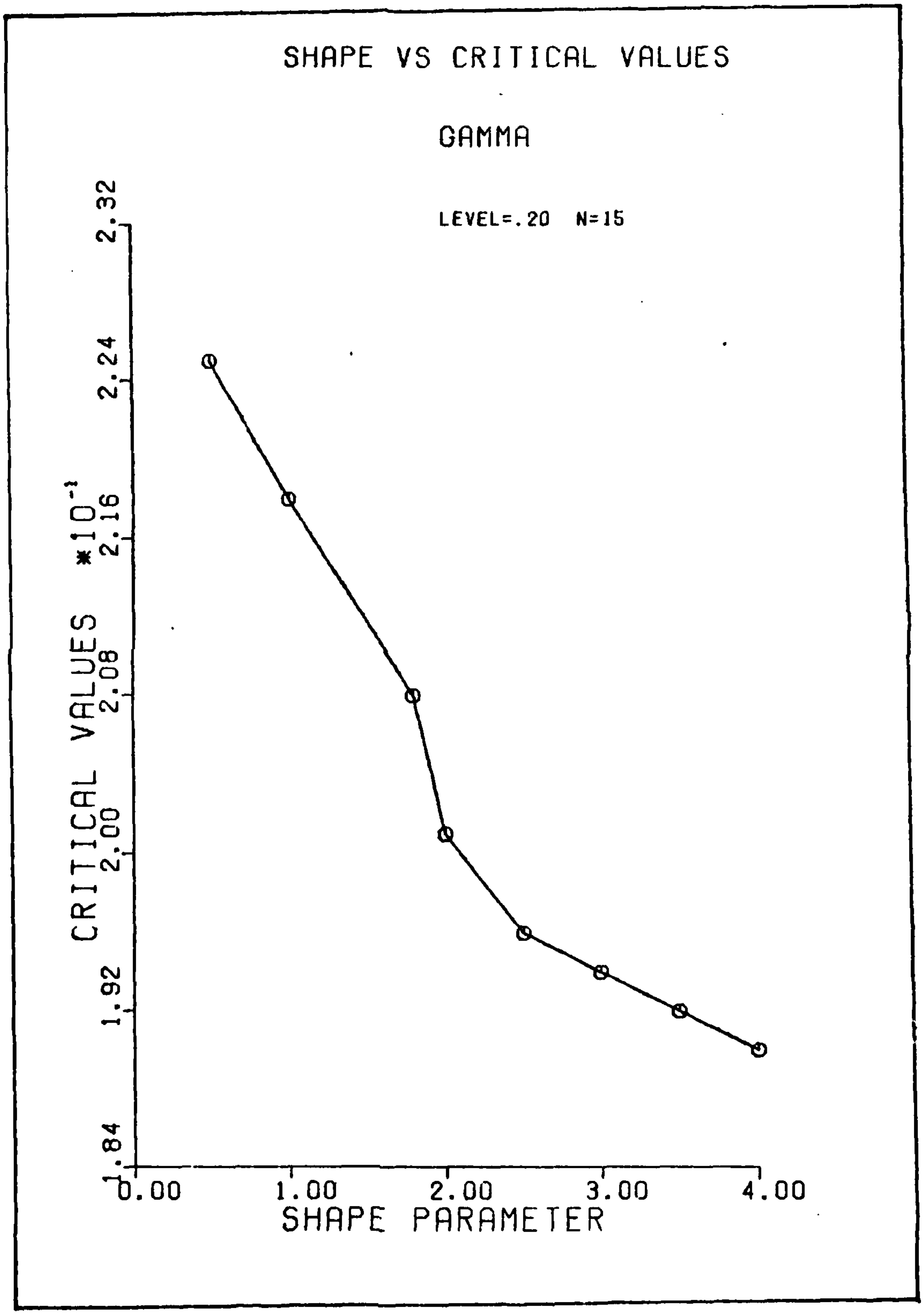


FIG 33. Shape vs K-S Critical Values, Level = .20, n = 15

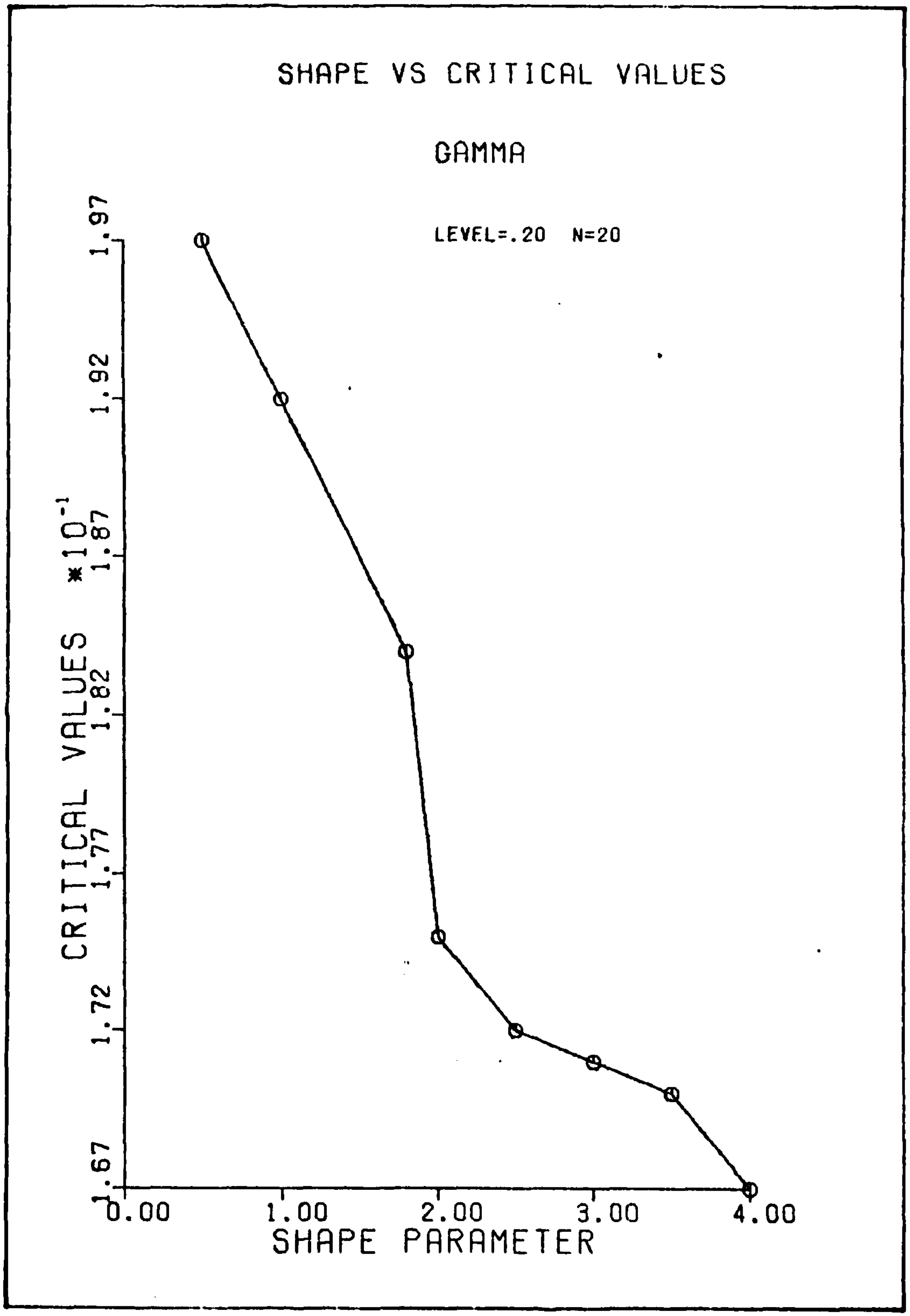


FIG 34. Shape vs K-S Critical Values, Level = .20, n = 20

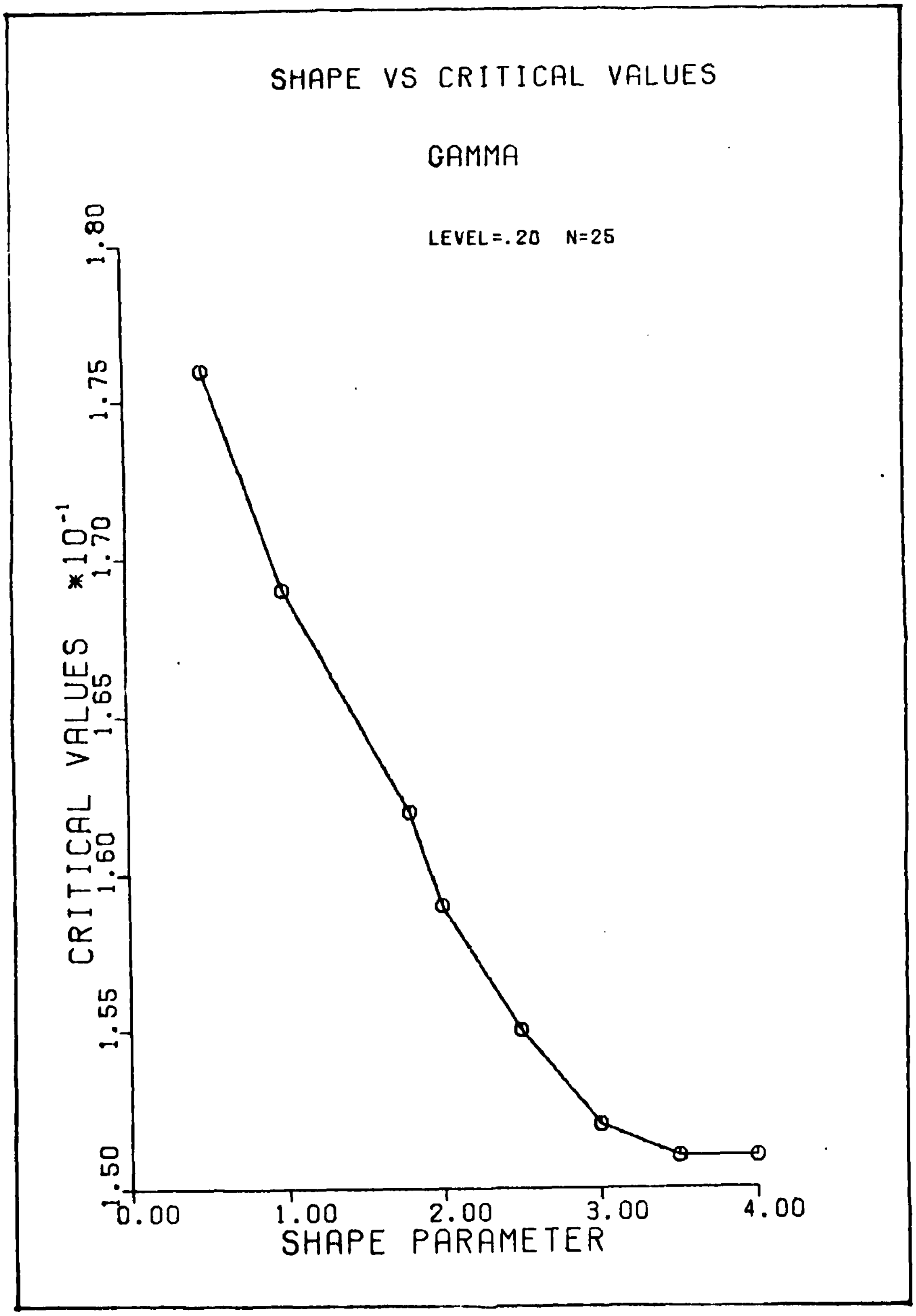


FIG 35. Shape vs K-S Critical Values, Level = .20, n = 25

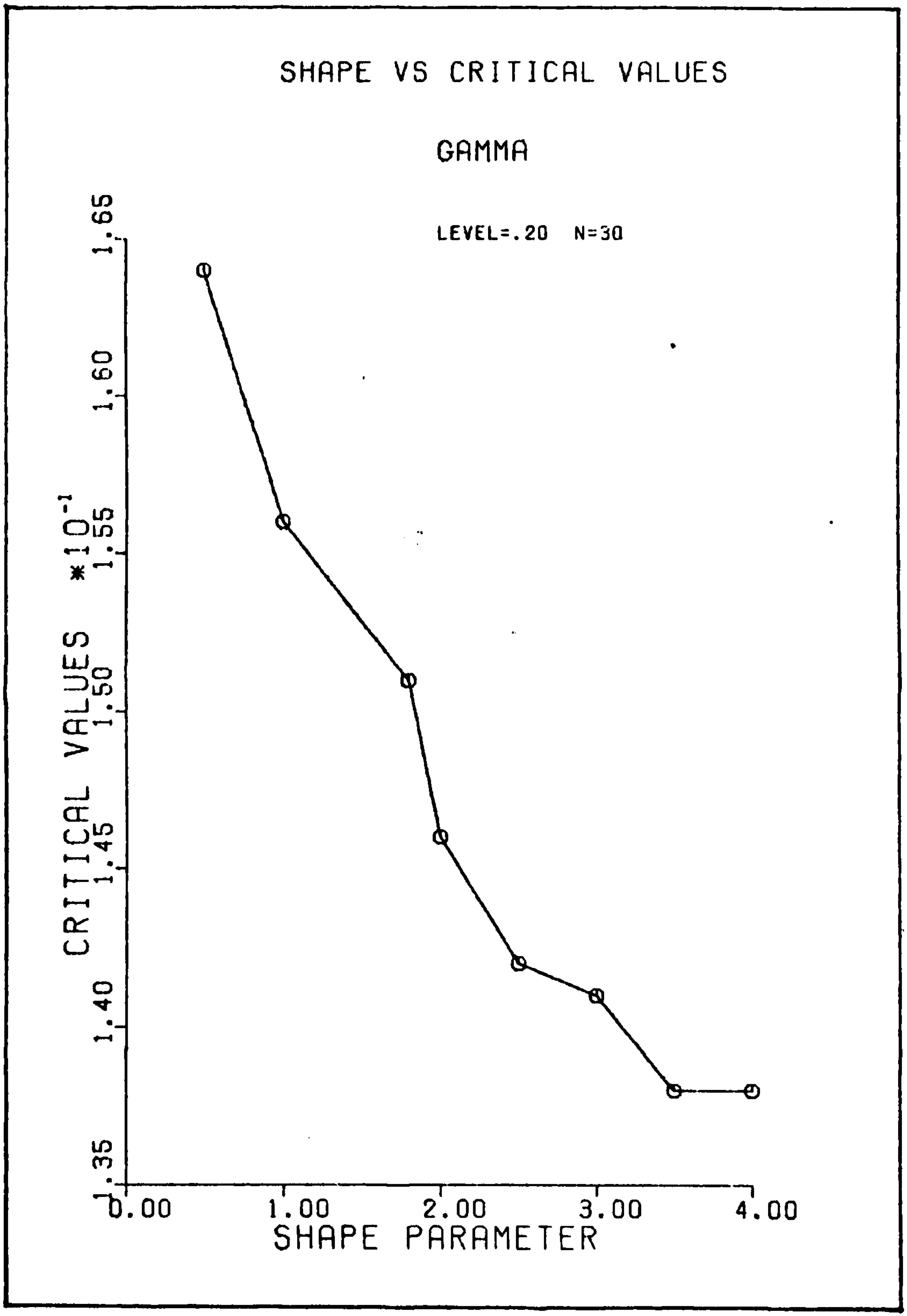


FIG 36. Shape vs K-S Critical Values, Level = .20, n = 30

APPENDIX E

Graphs of the Anderson-Darling

Critical Values Verses the

Gamma Shape Parameters

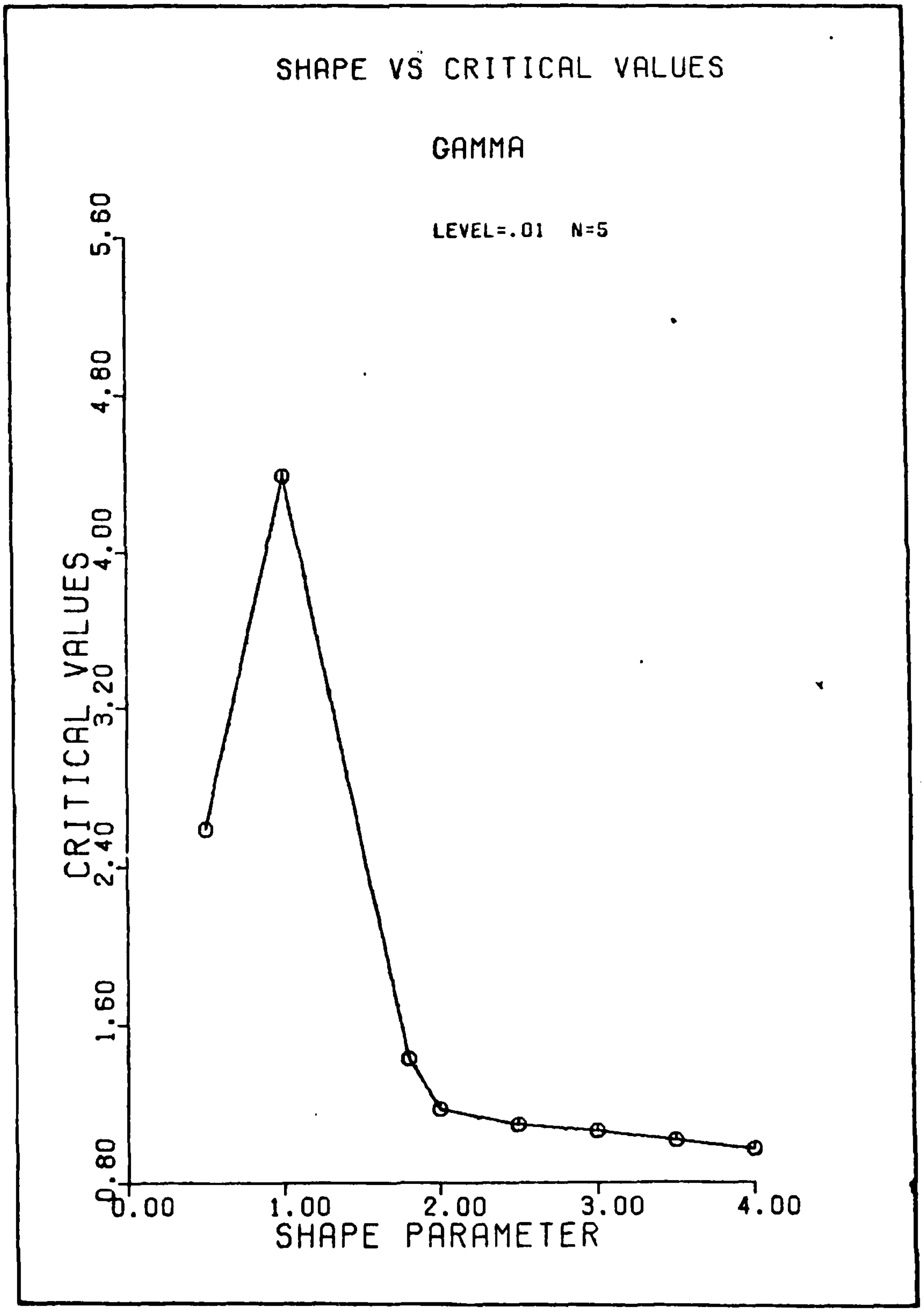


FIG 37. Shape vs A^2 Critical Values, Level = .01, n = 5

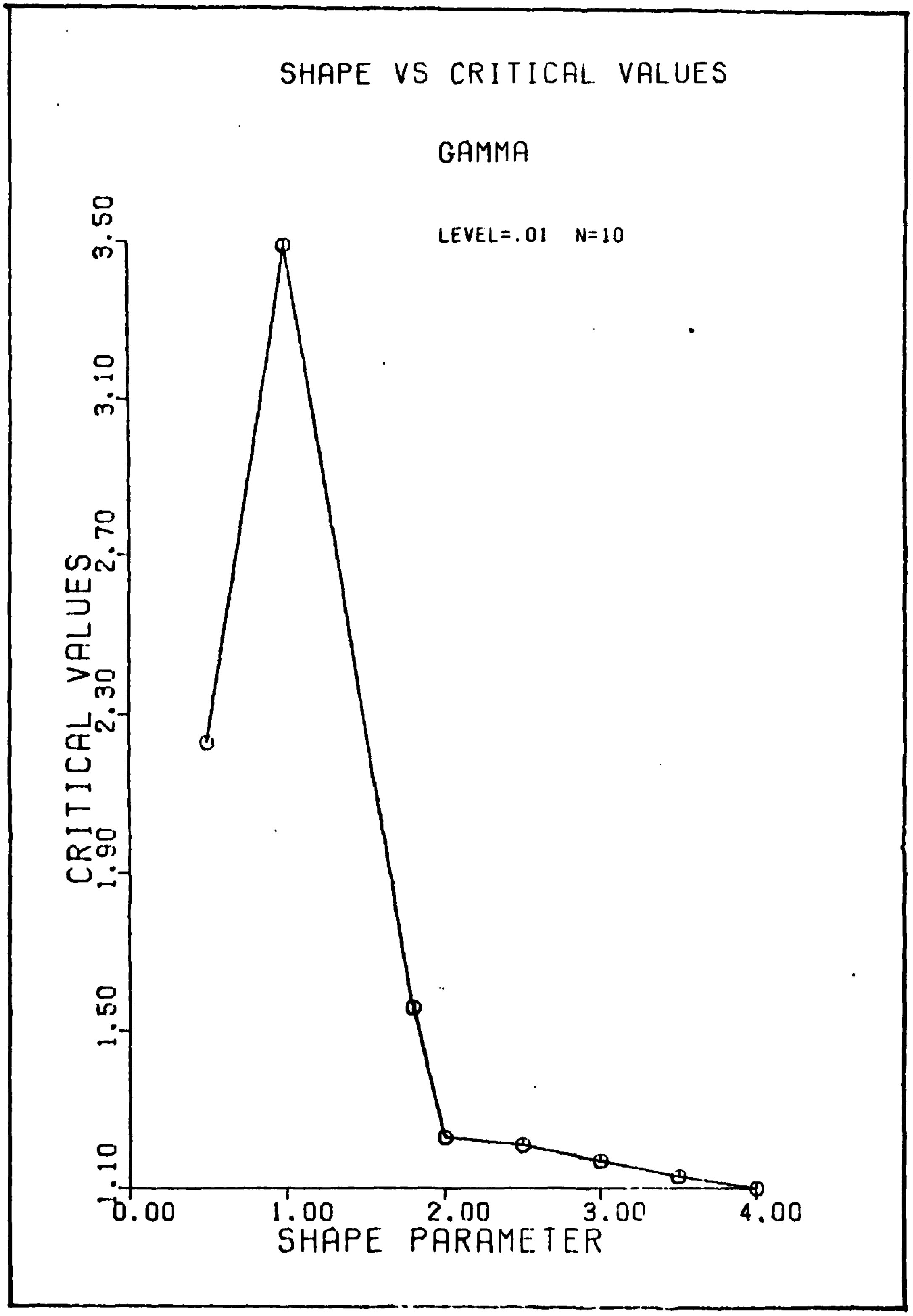


FIG 38. Shape vs A^2 Critical Values, Level = .01, n = 10

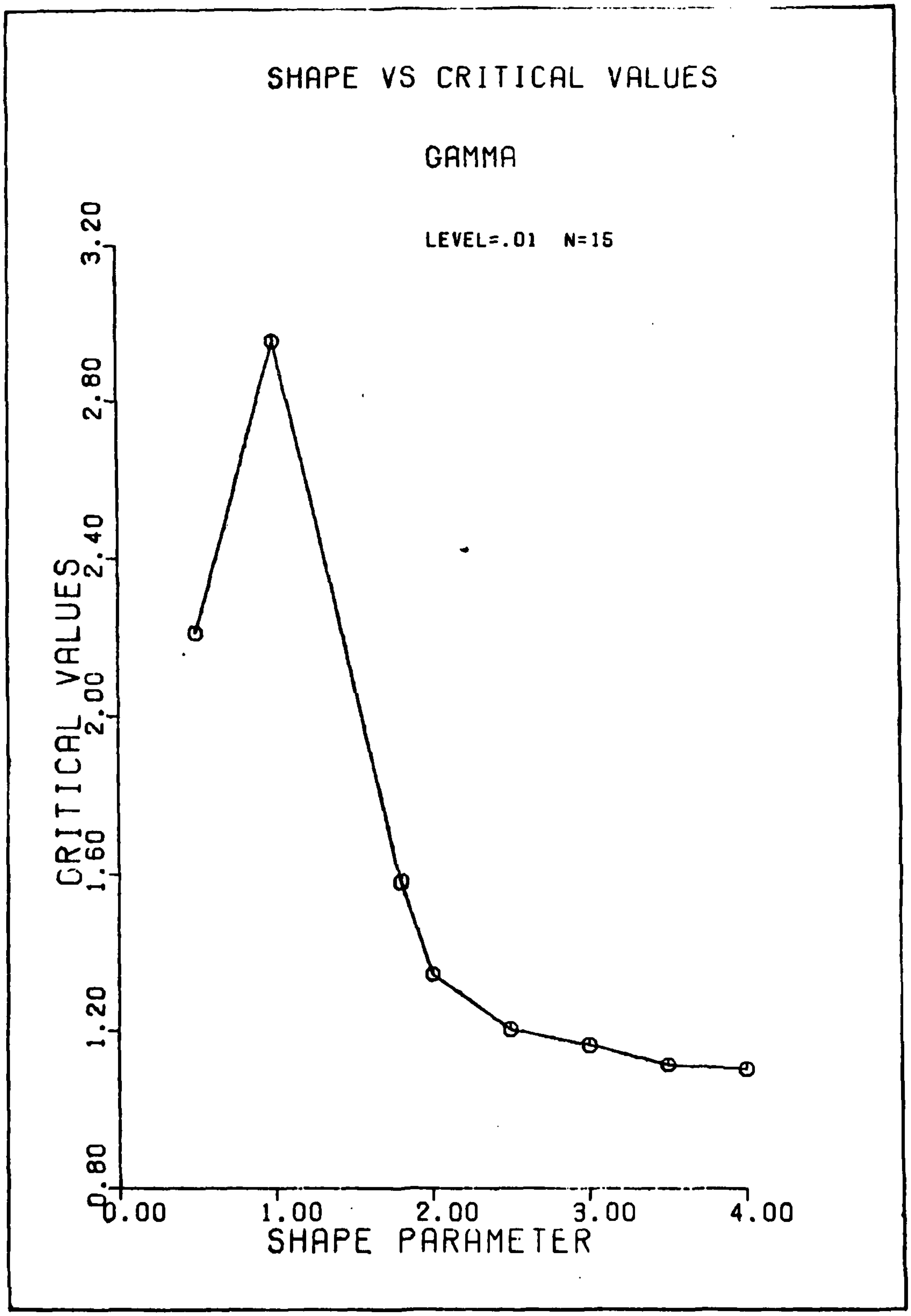


FIG 39. Shape vs A^2 Critical Values, Level = .01, n = 15

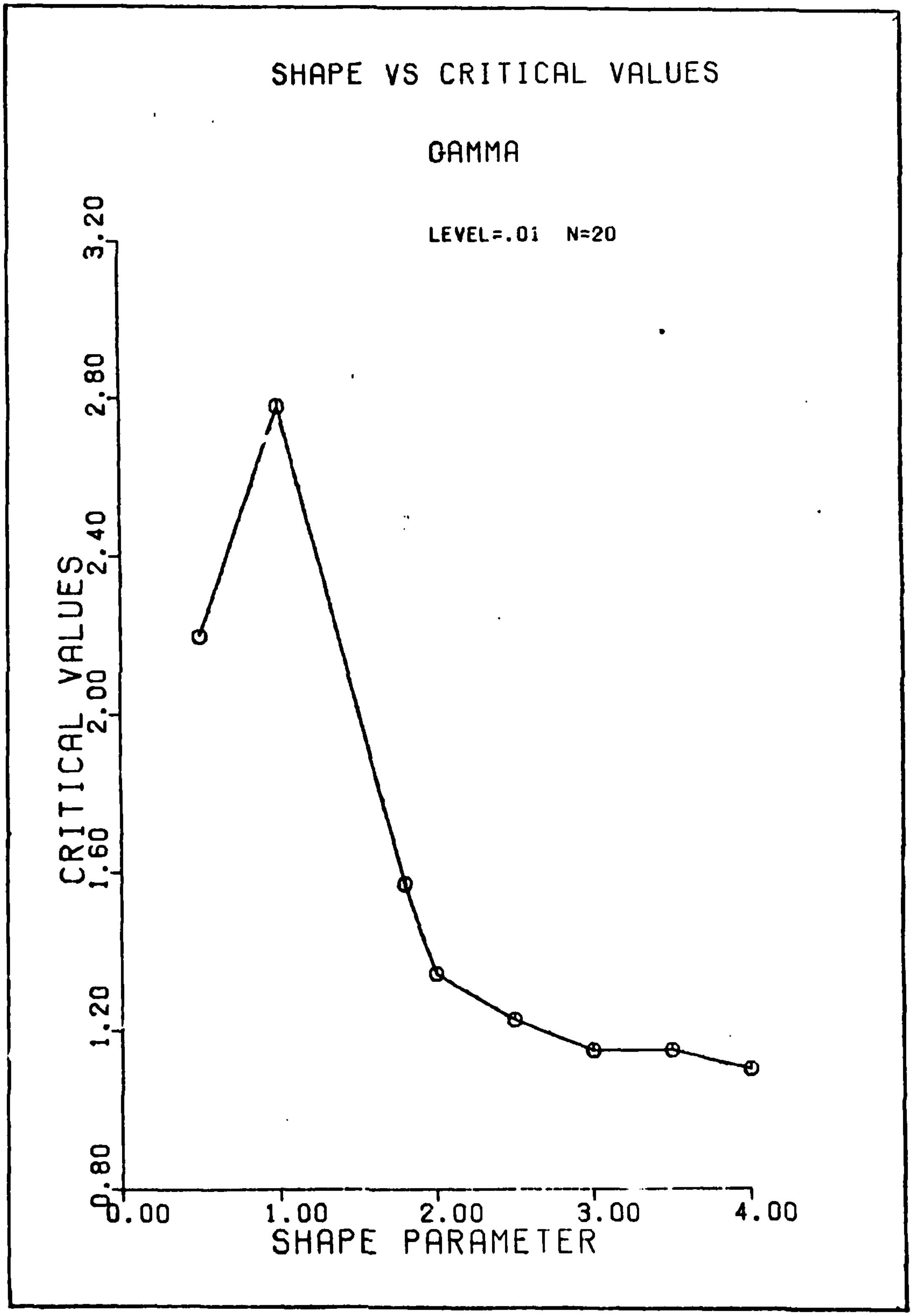


FIG 48. Shape vs A^2 Critical Values, Level = .01, n = 20

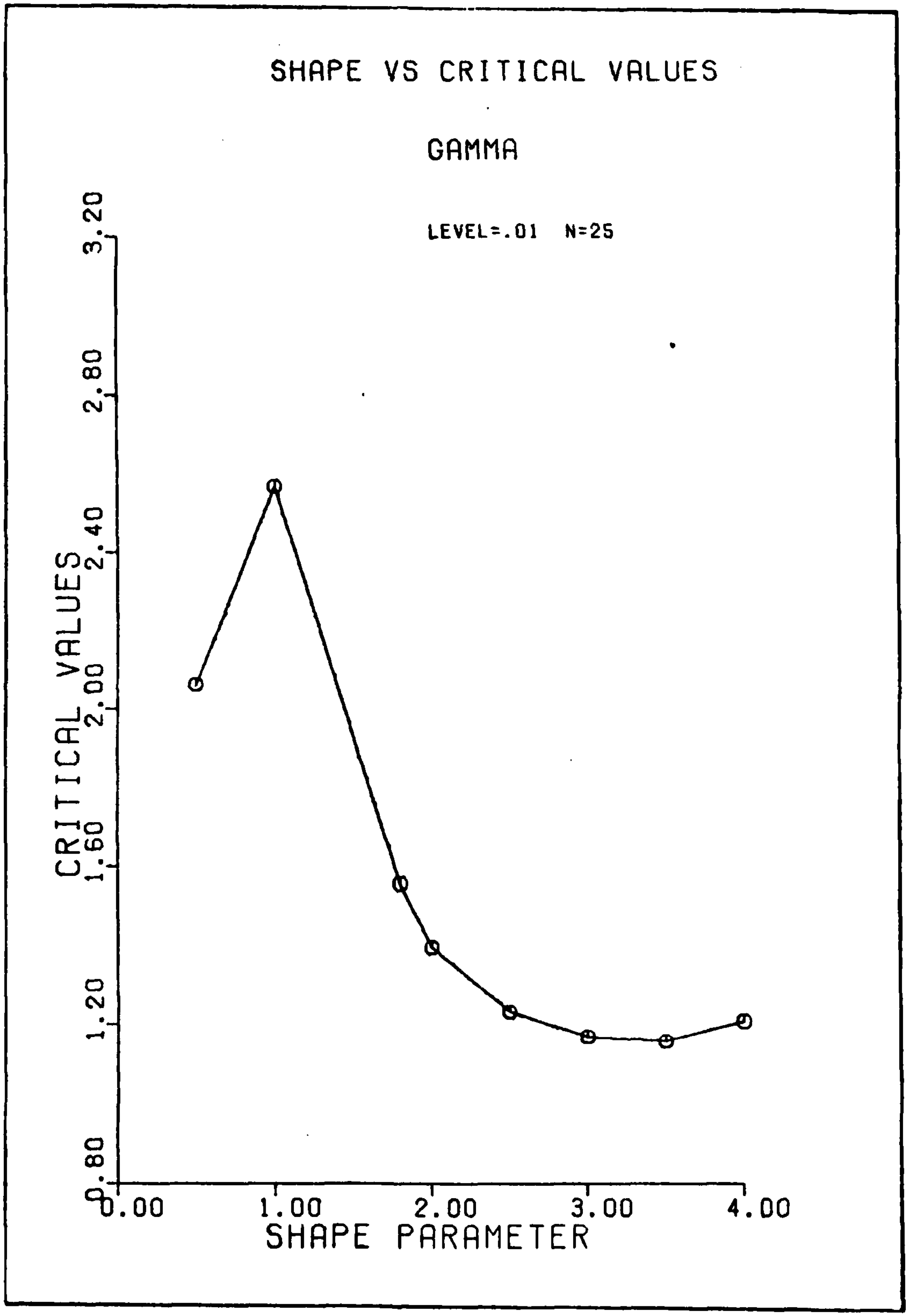


FIG 41. Shape vs A^2 Critical Values, Level = .01, n = 25

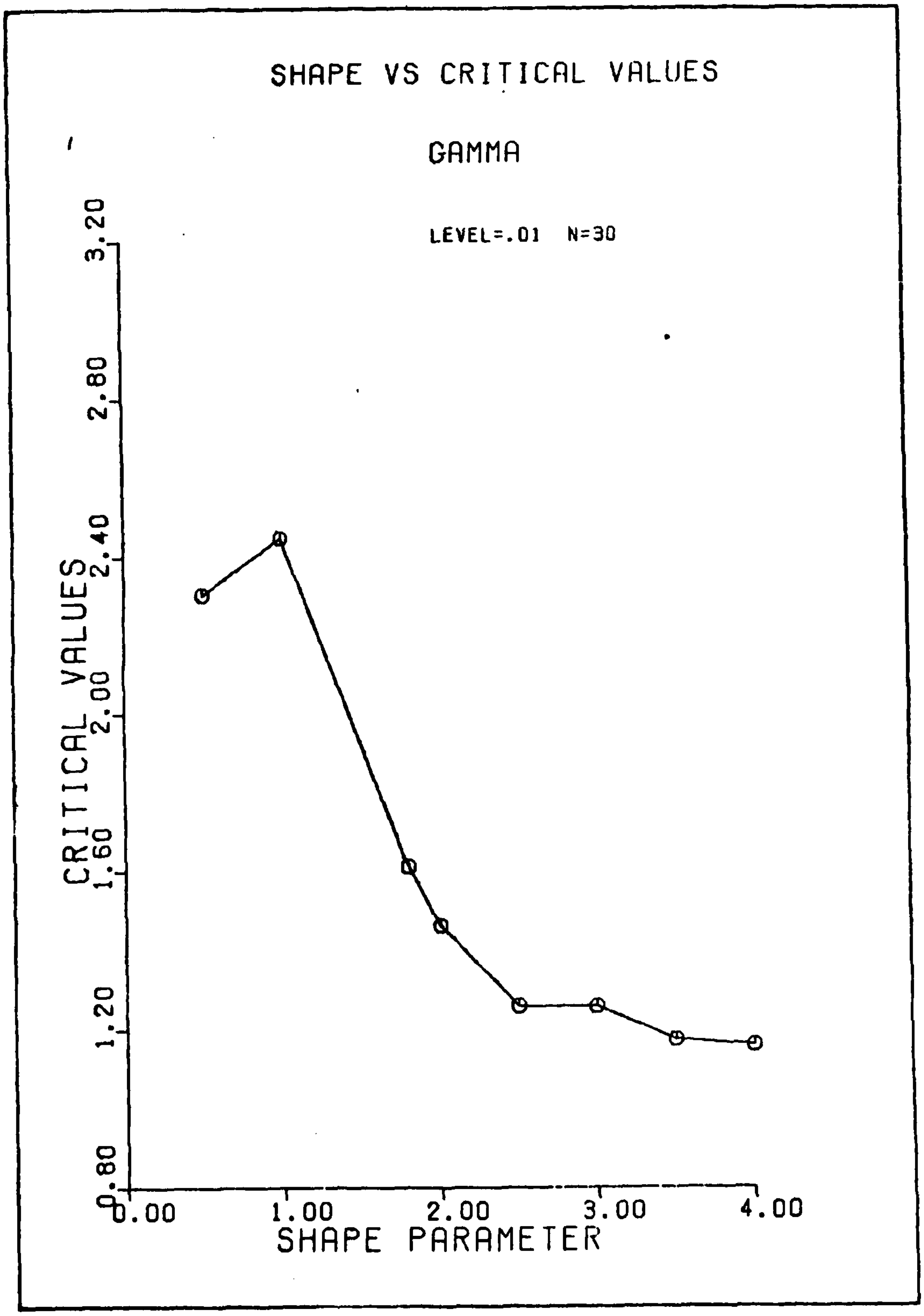


FIG 42. Shape vs A^2 Critical Values, Level = .01, n = 30

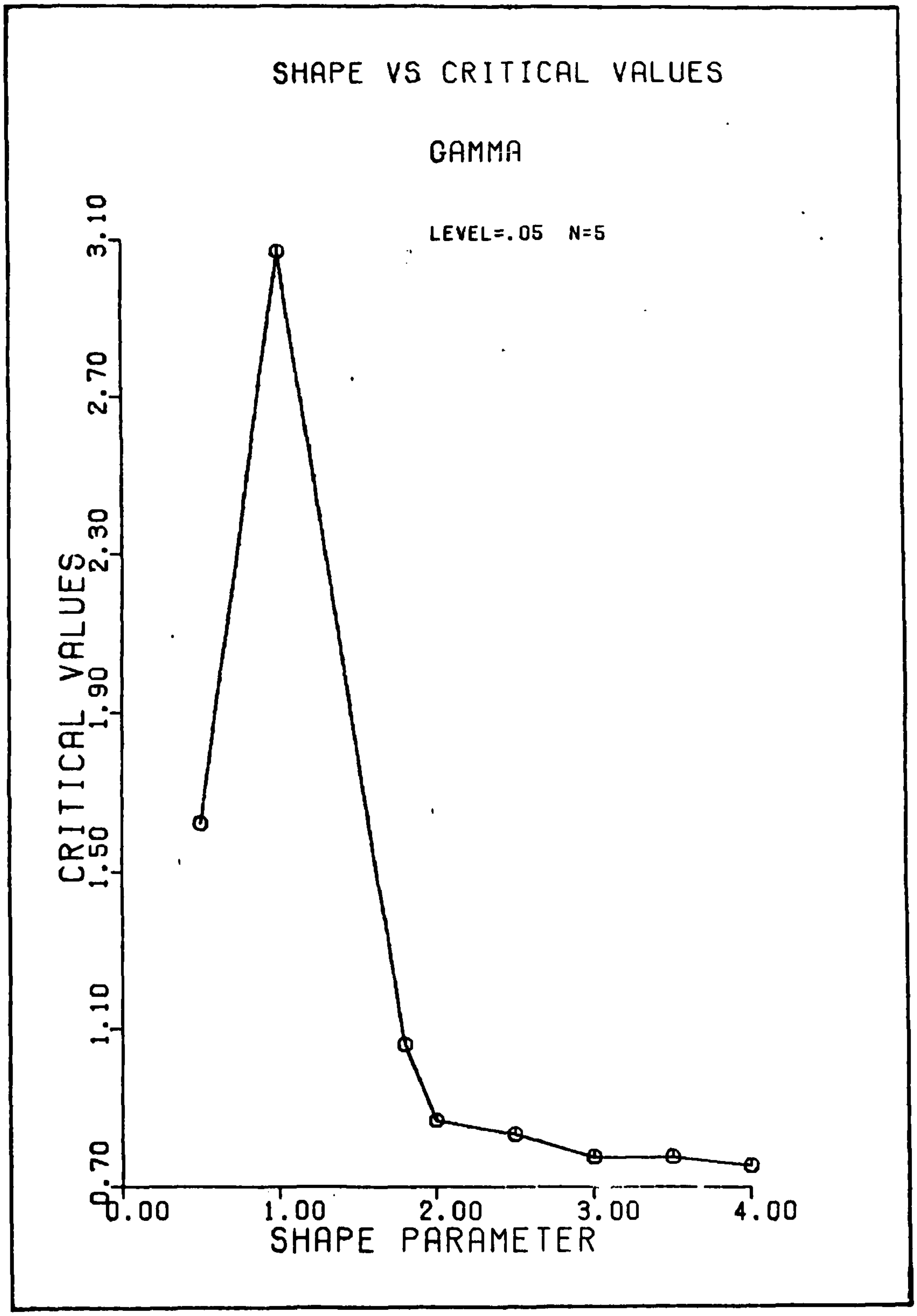


FIG 43. Shape vs A^2 Critical Values, Level = .05, n = 5

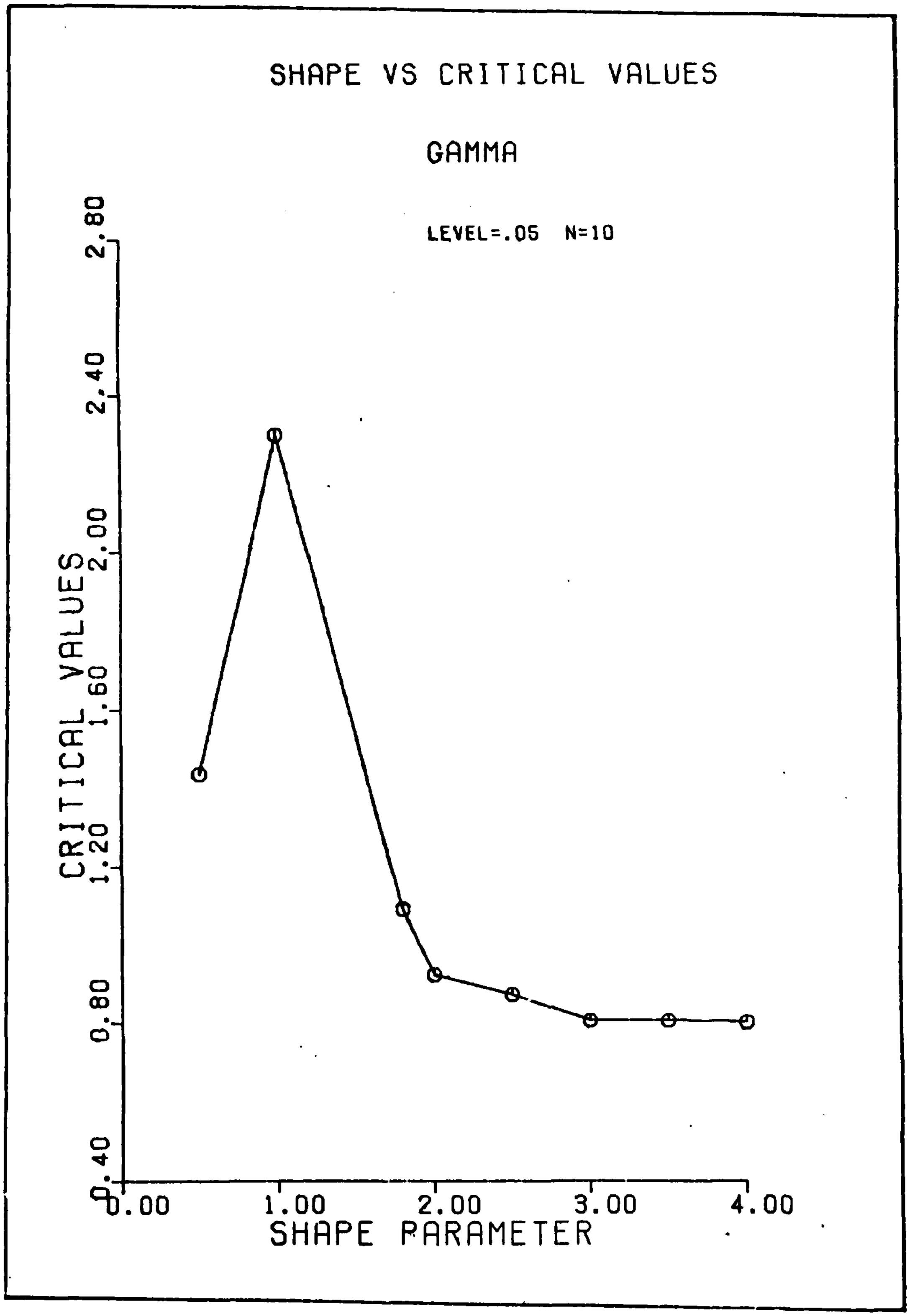


FIG 44. Shape vs A^2 Critical Values, Level = .05, n = 10

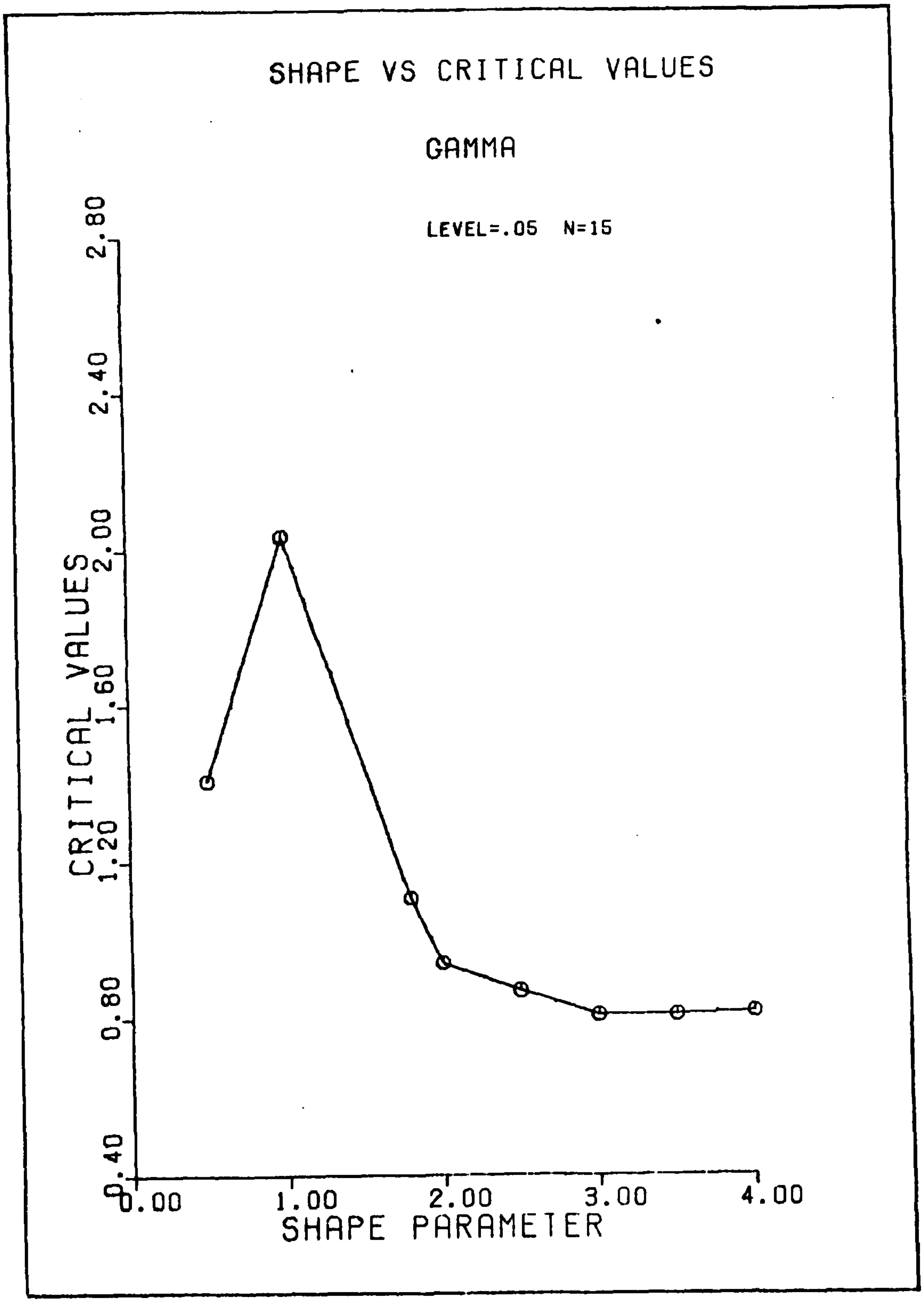


FIG 45. Shape vs A^2 Critical Values, Level = .05, n = 15

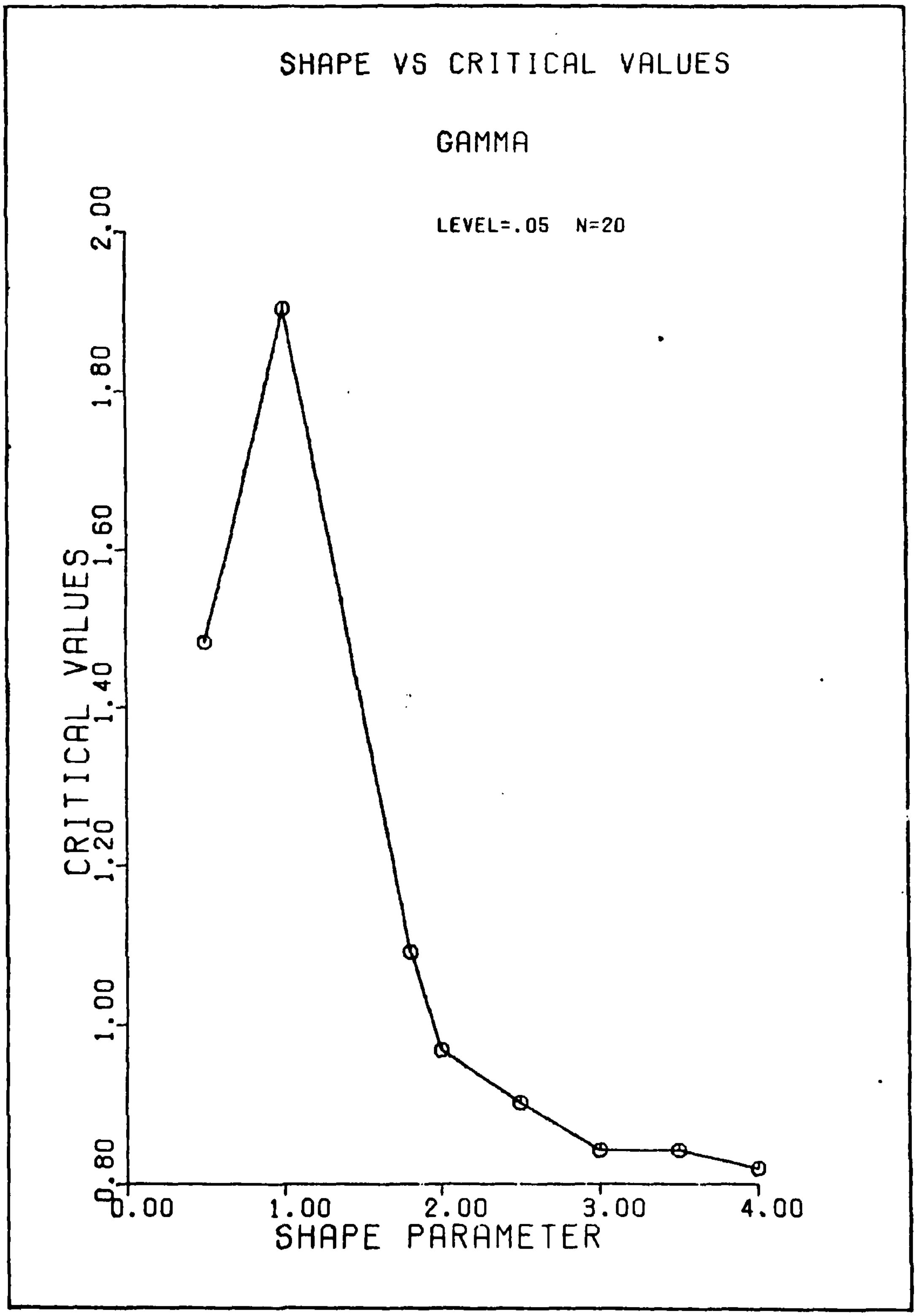


FIG 45. Shape vs A^2 Critical Values, Level = .05, n = 20

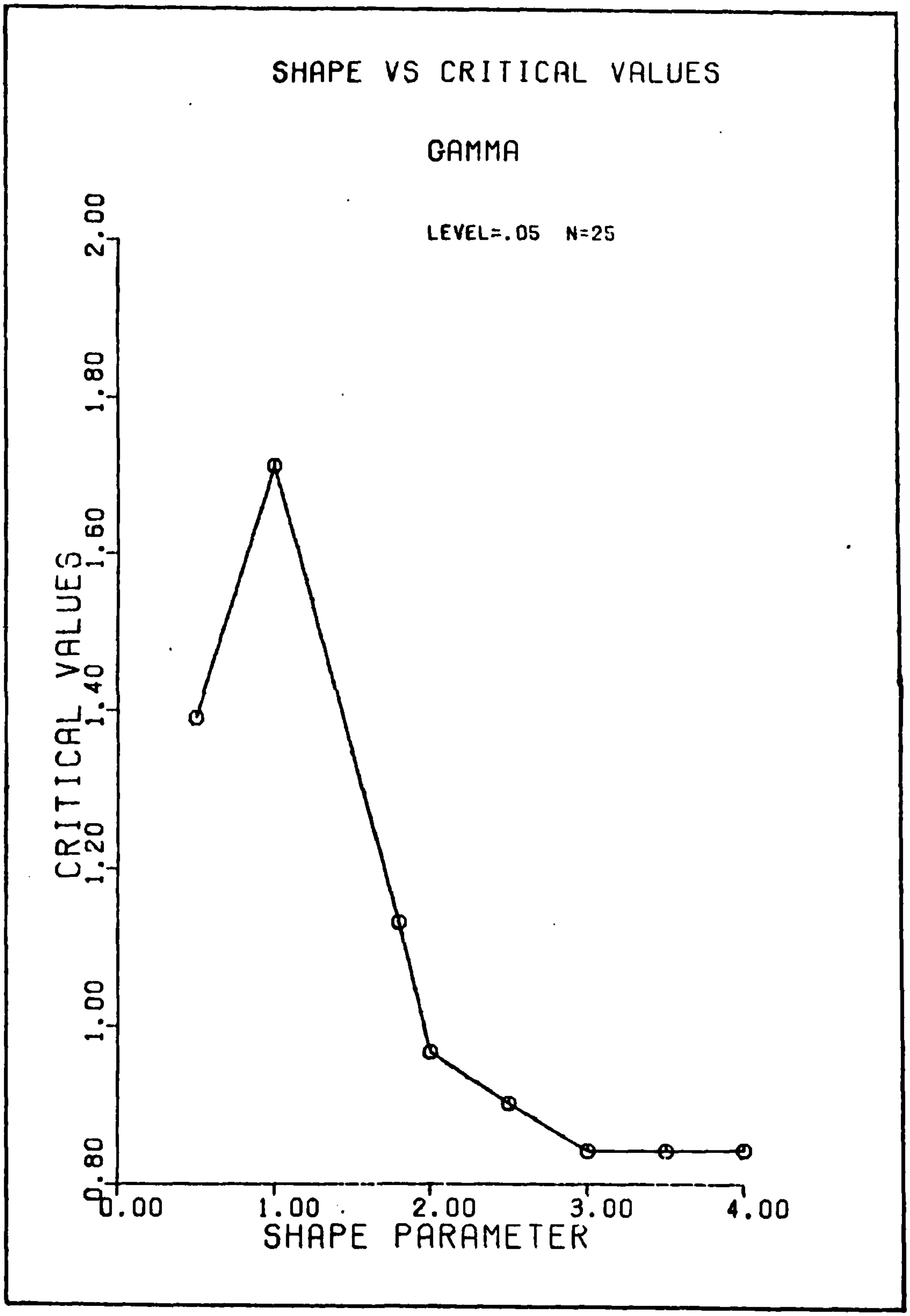


FIG 47. Shape vs A^2 Critical Values, Level = .05, n = 25

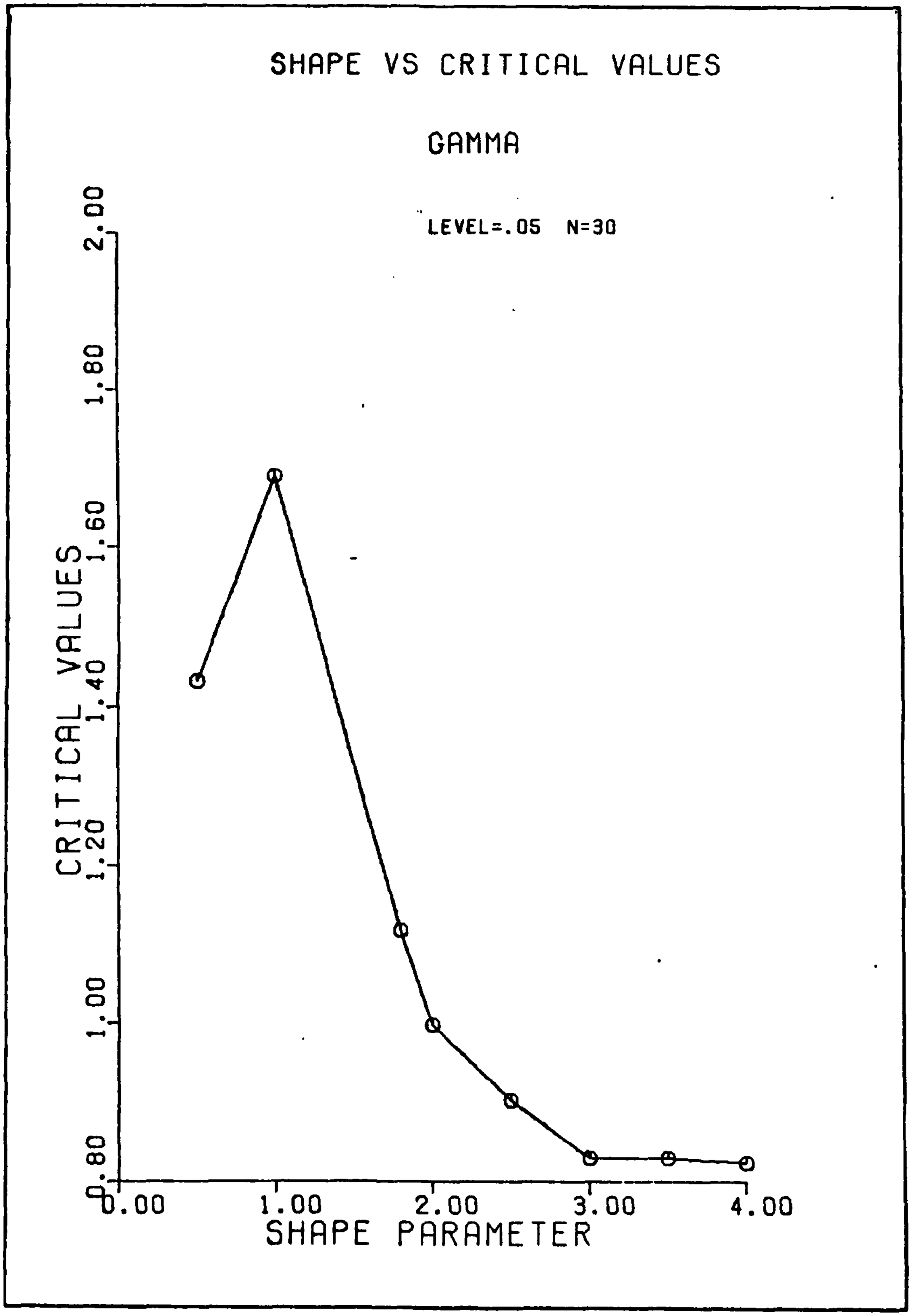


FIG 48. Shape vs A^2 Critical Values, Level = .05, n = 30

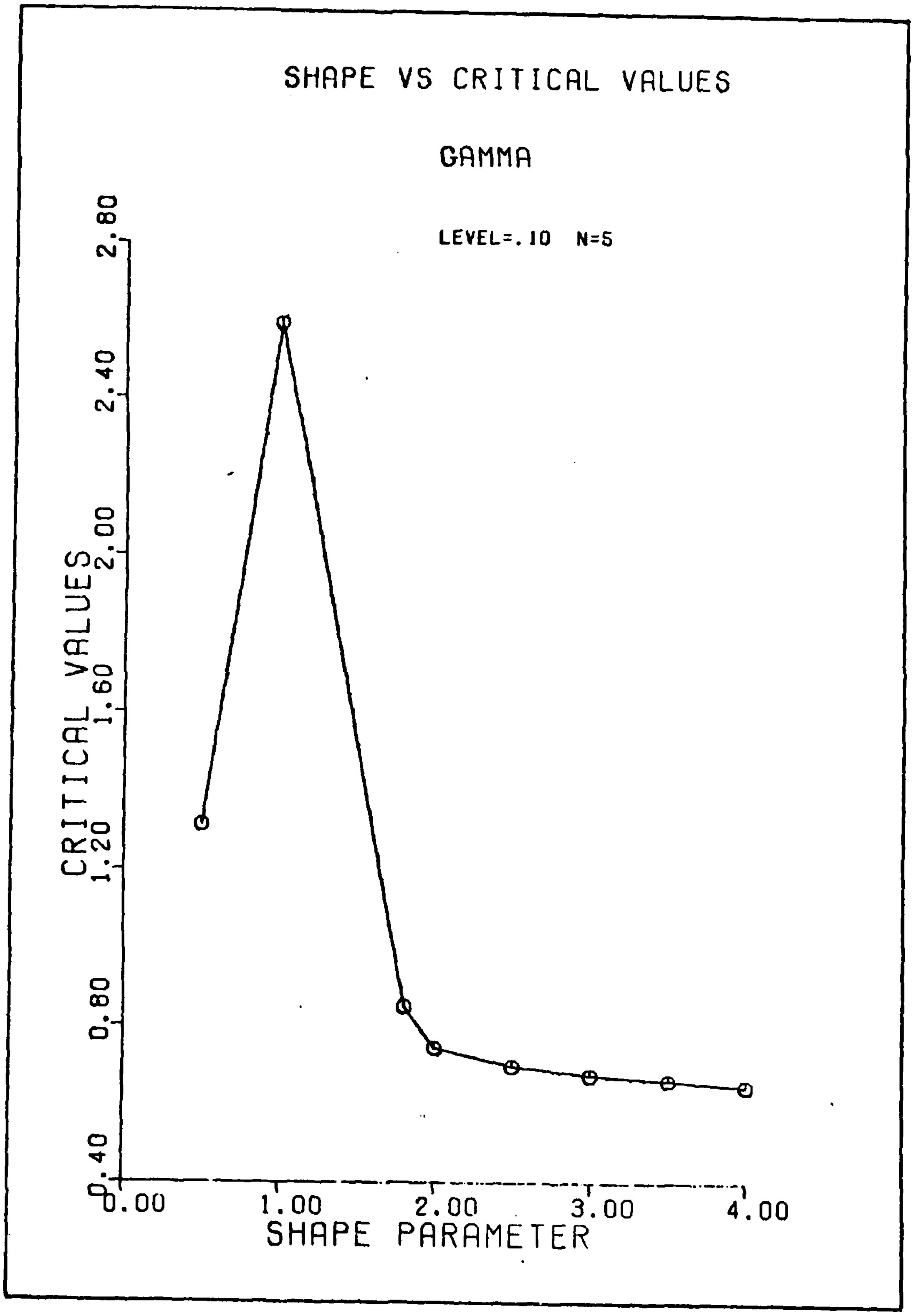


FIG 45. Shape vs A^2 Critical Values, Level = .10, n = 5

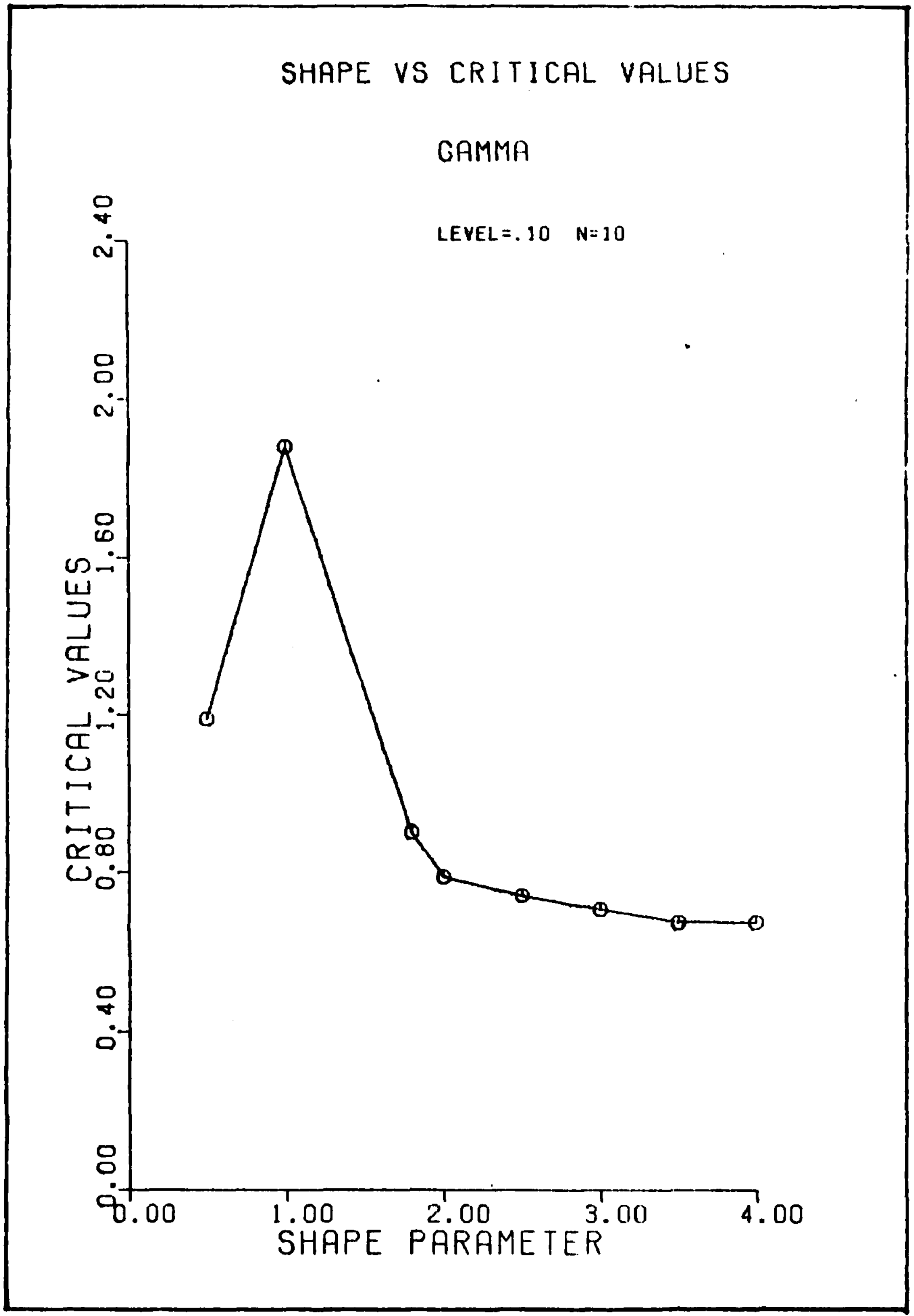


FIG 58. Shape vs A^2 Critical Values, Level = .10, n = 10

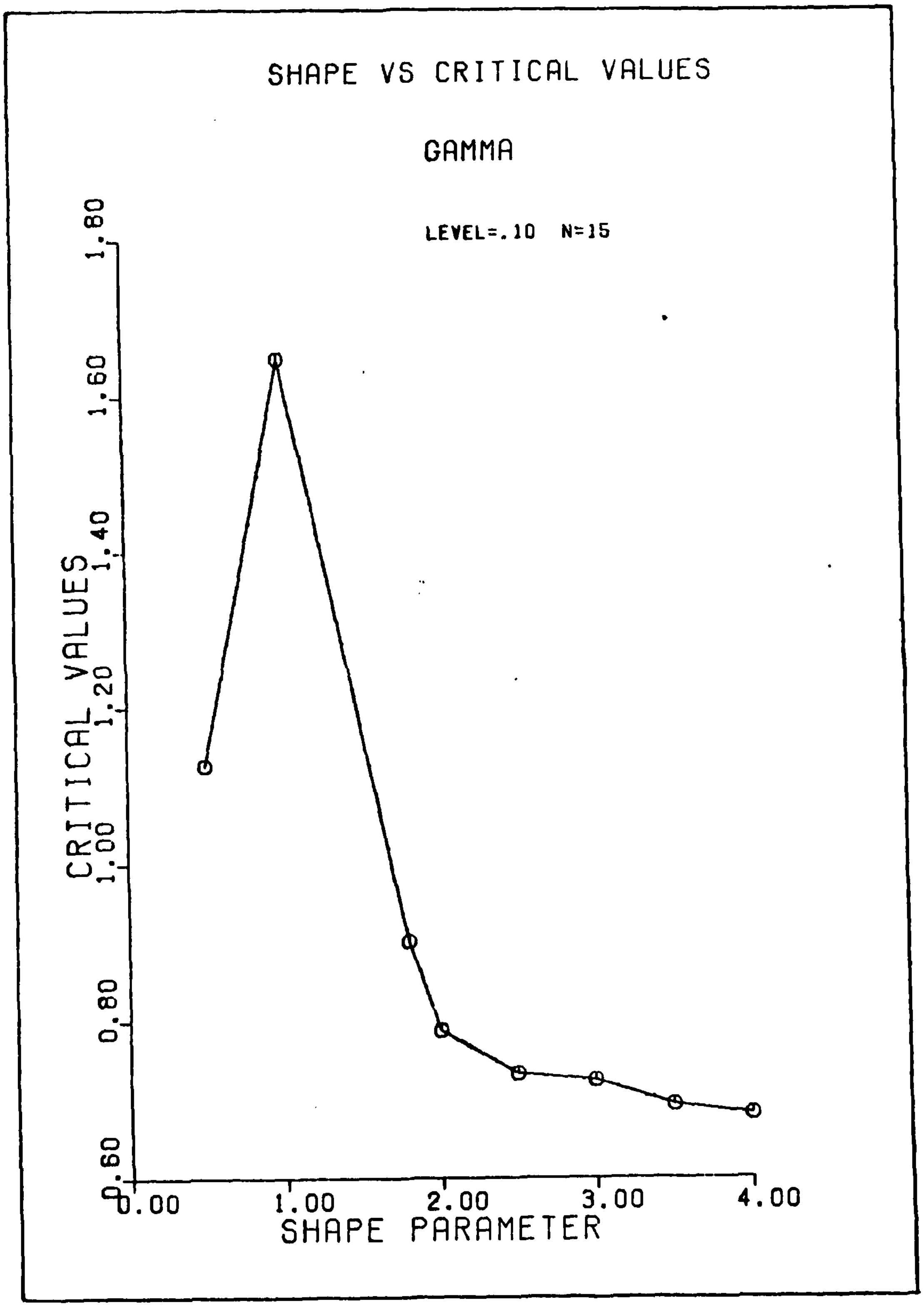


FIG 51. Shape vs A^2 Critical Values, Level = .10, n = 15

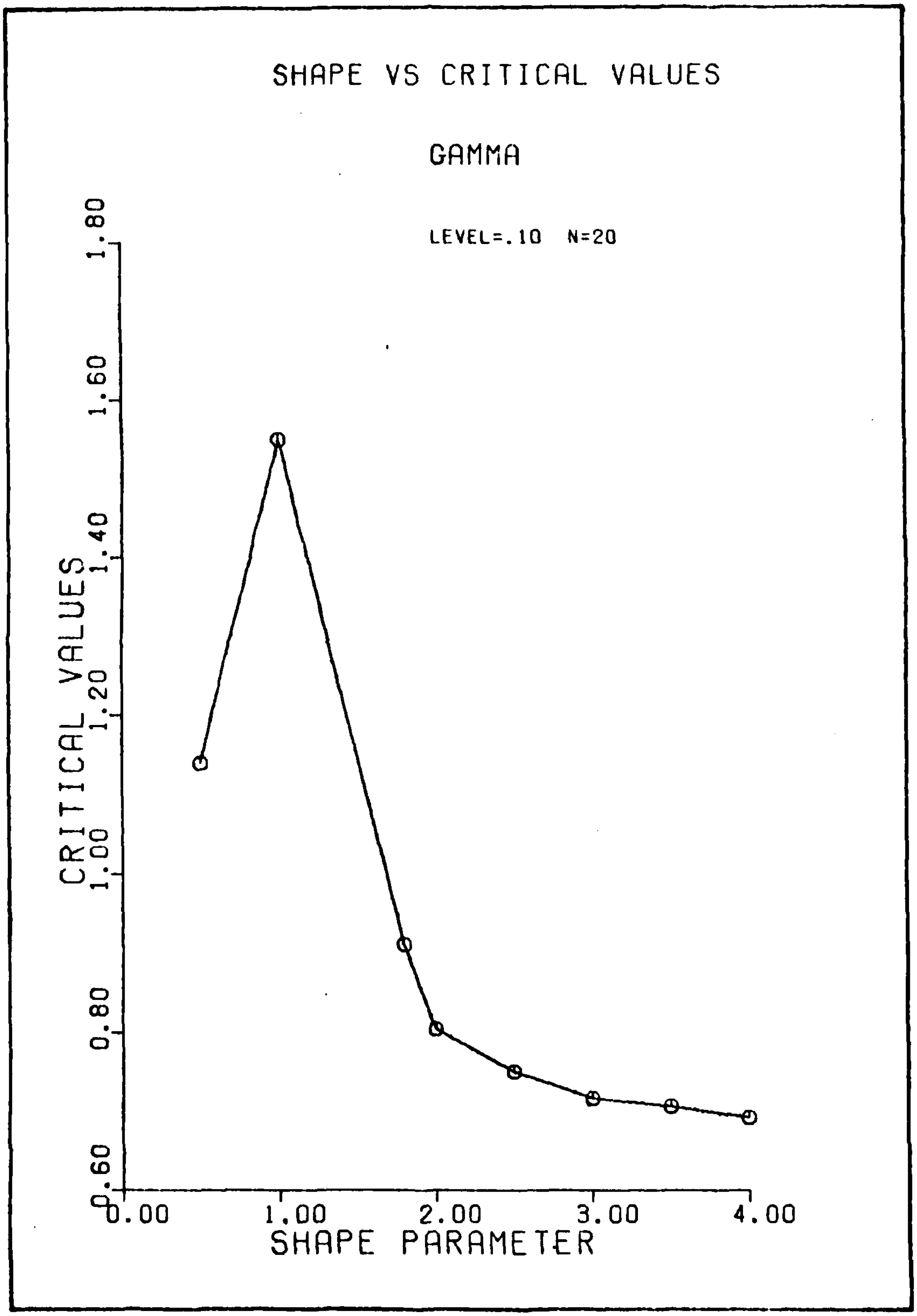


FIG 52. Shape vs A^2 Critical Values, Level = .10, n = 20

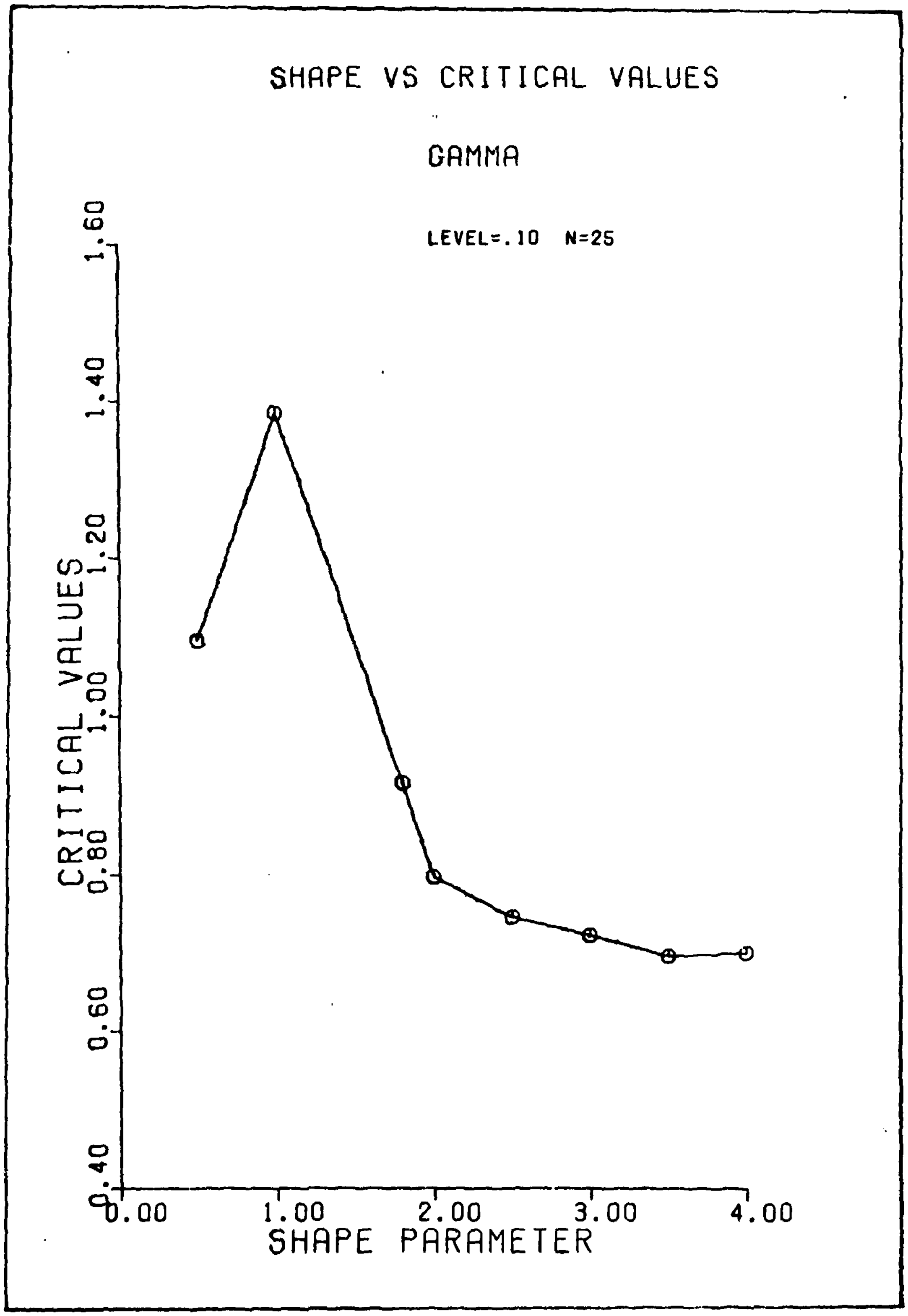


FIG 53. Shape vs A^2 Critical Values, Level = .10, n = 25

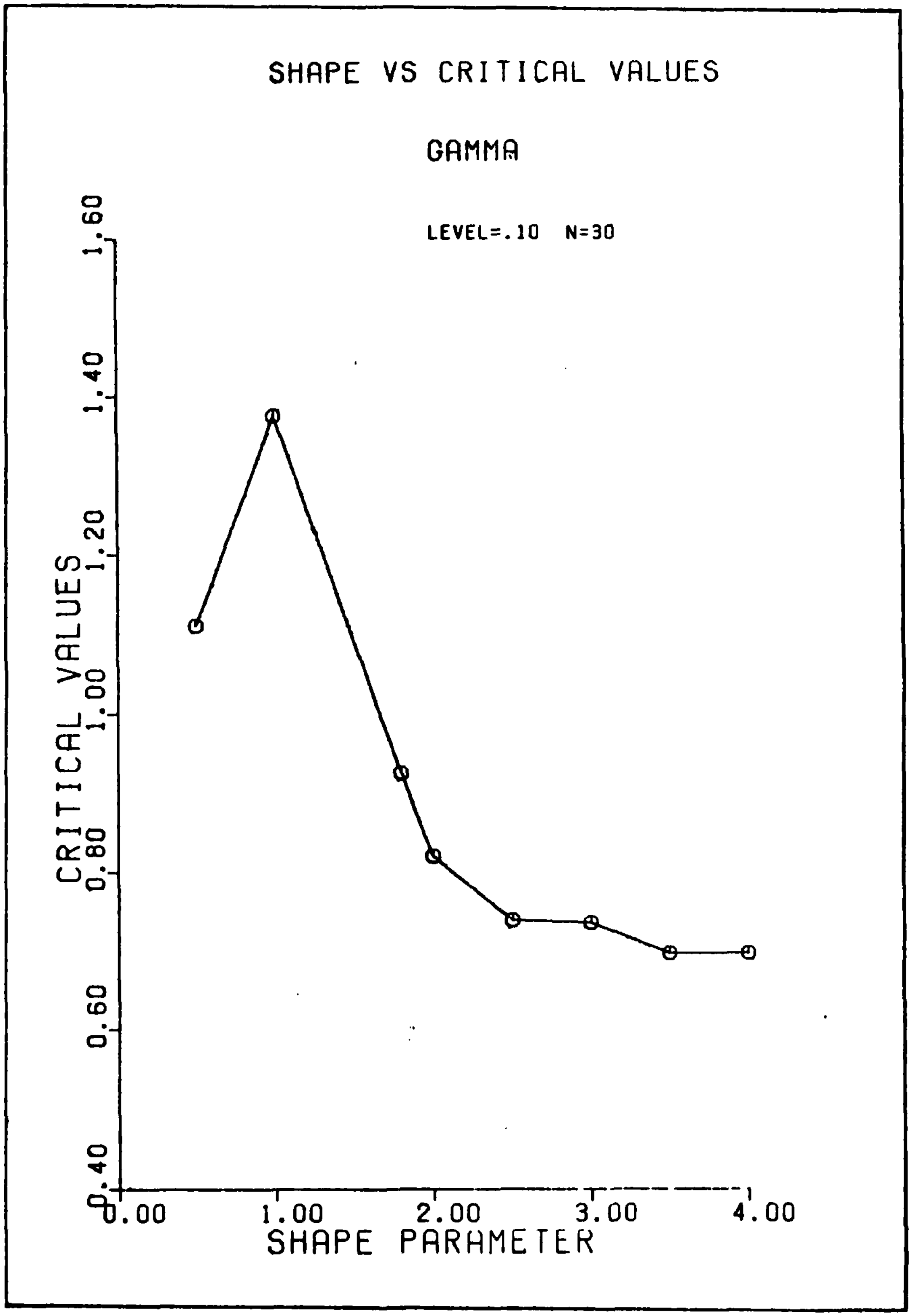


FIG 54. Shape vs A^2 Critical Values, Level = .10, n = 30

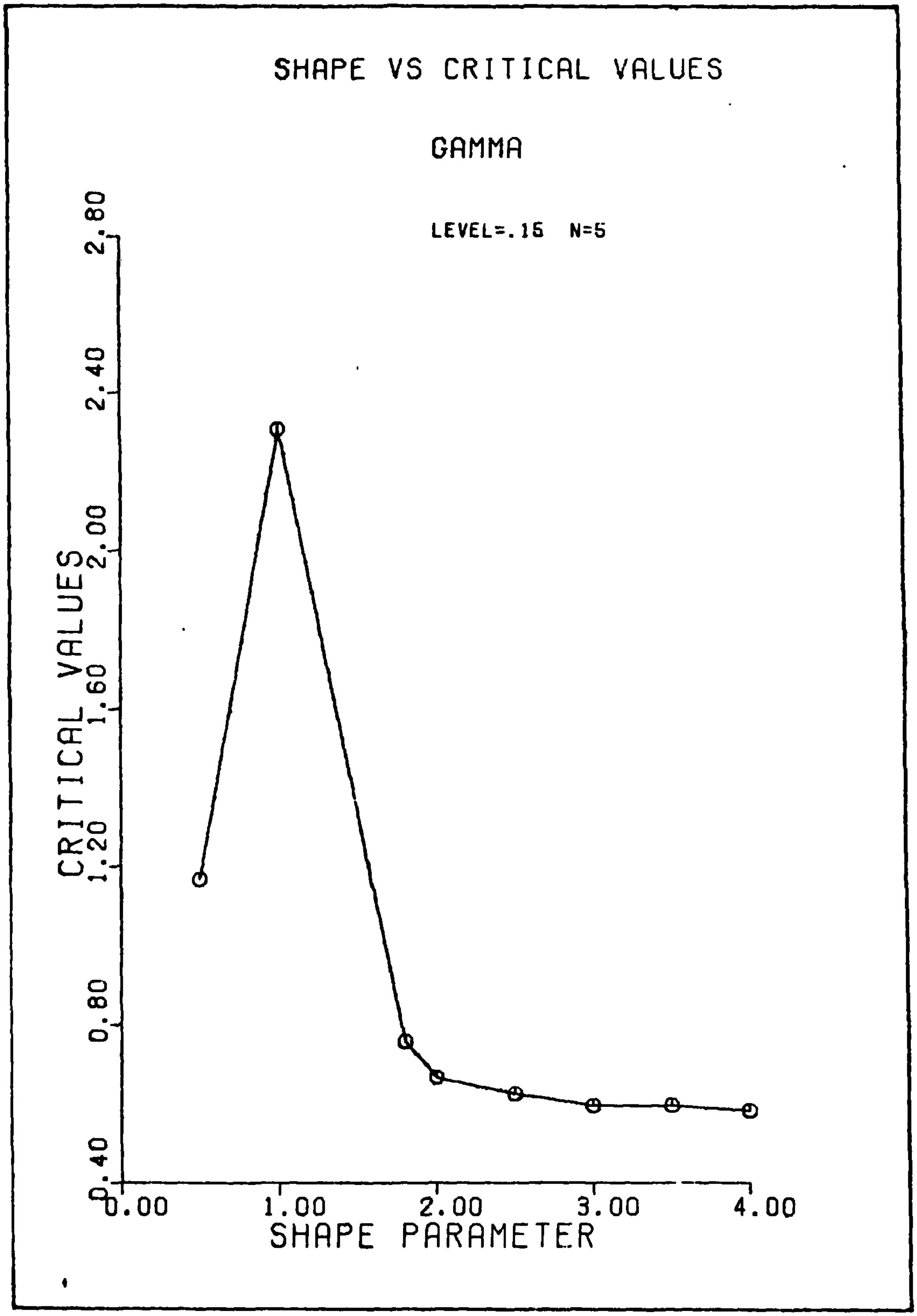


FIG 55. Shape vs A^2 Critical Values, Level = .15, n = 5

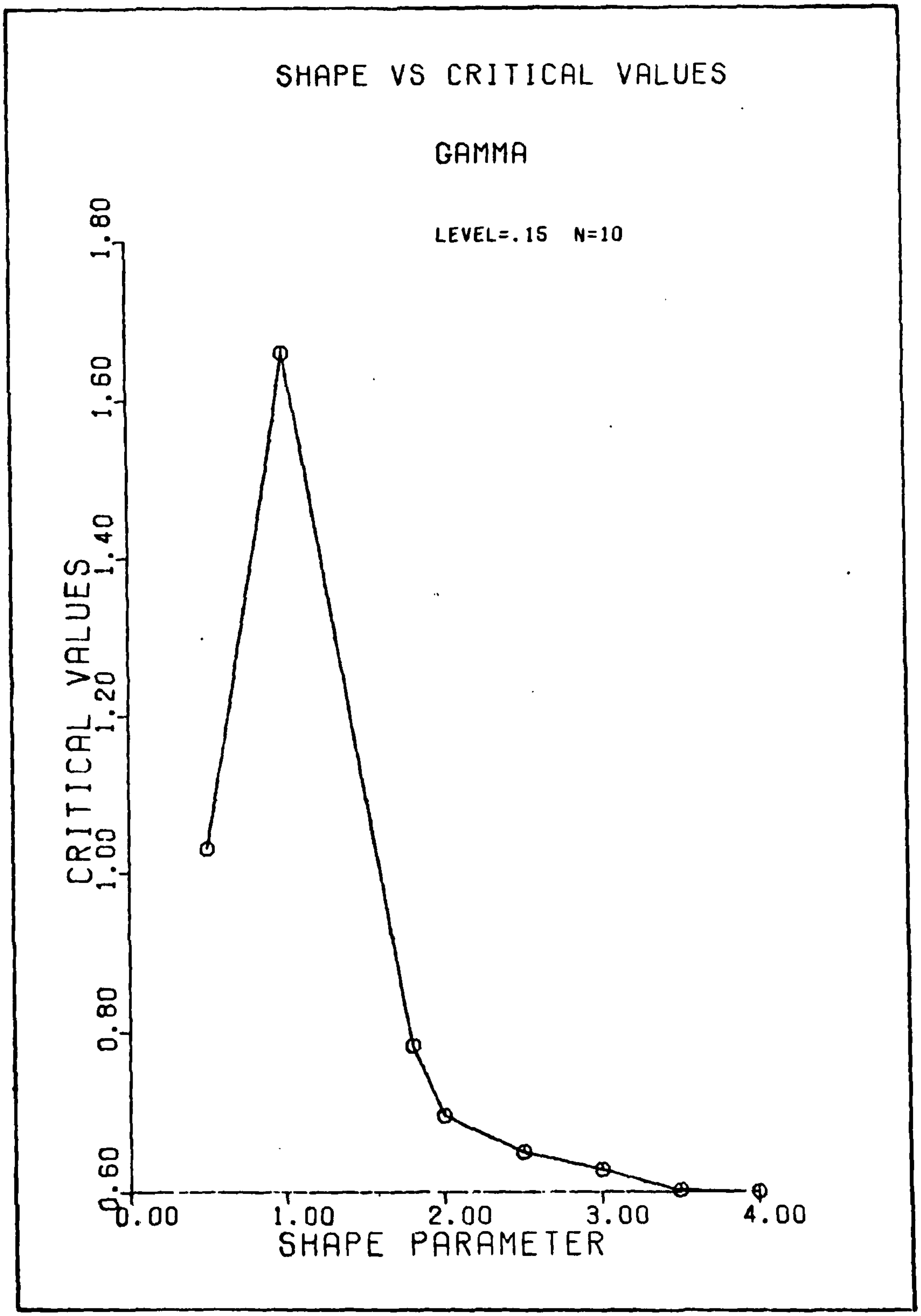


FIG 56. Shape vs A^2 Critical Values, Level = .15, n = 10

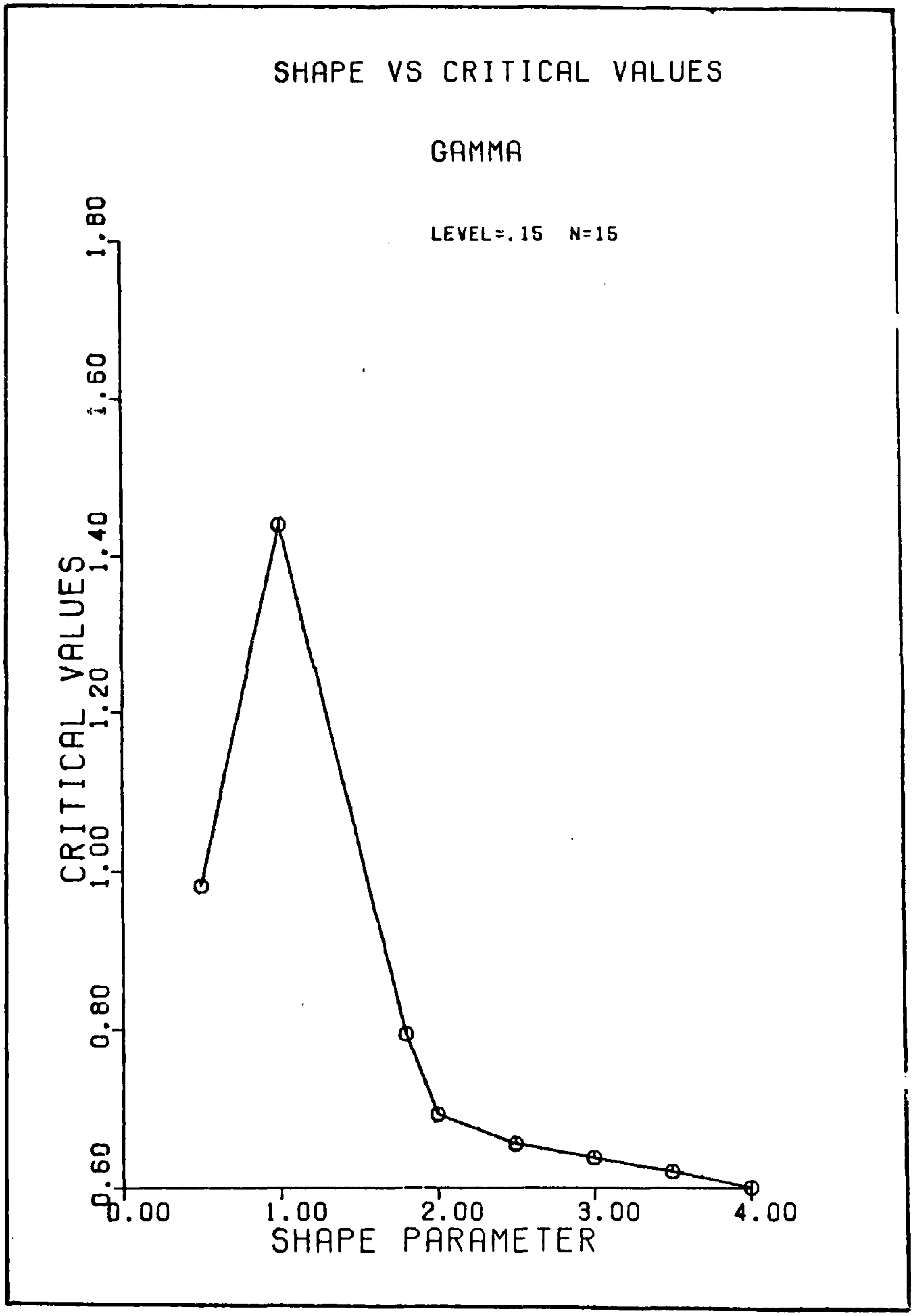


FIG 57. Shape vs A^2 Critical Values, Level = .15, n = 15

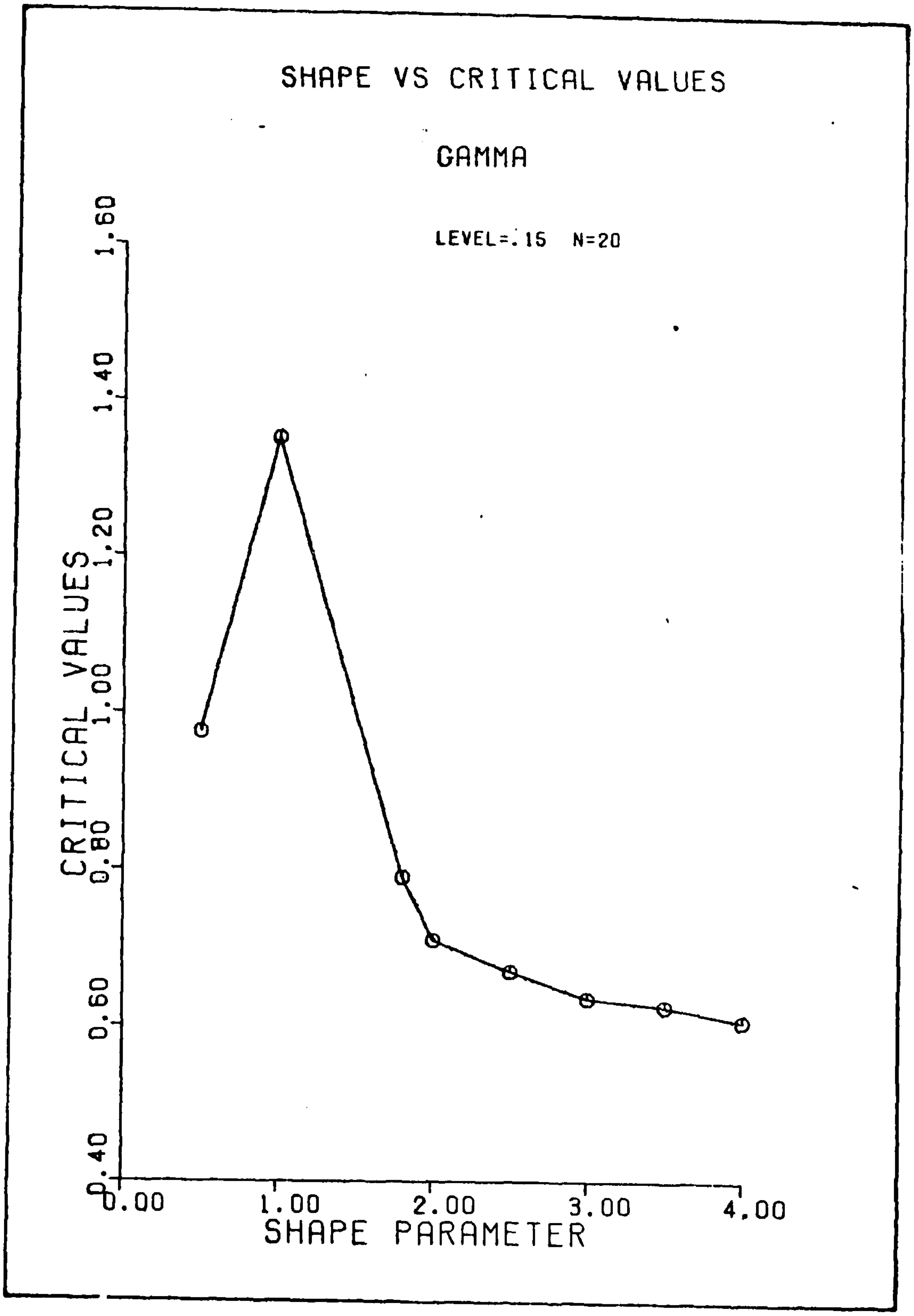


FIG 58. Shape vs A^2 Critical Values, Level = .15, n = 20

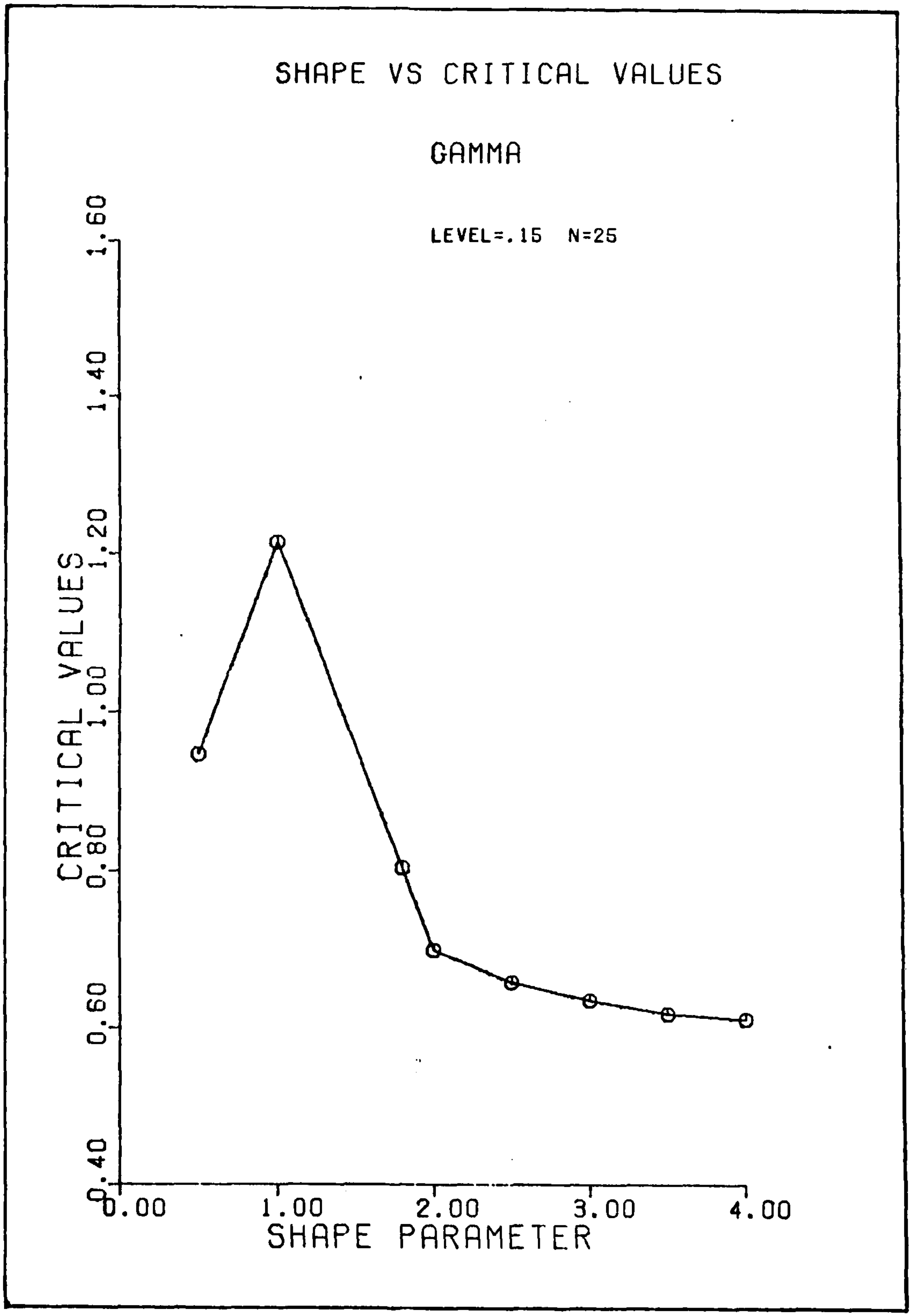


FIG 59. Shape vs A^2 Critical Values, Level = .15, n = 25

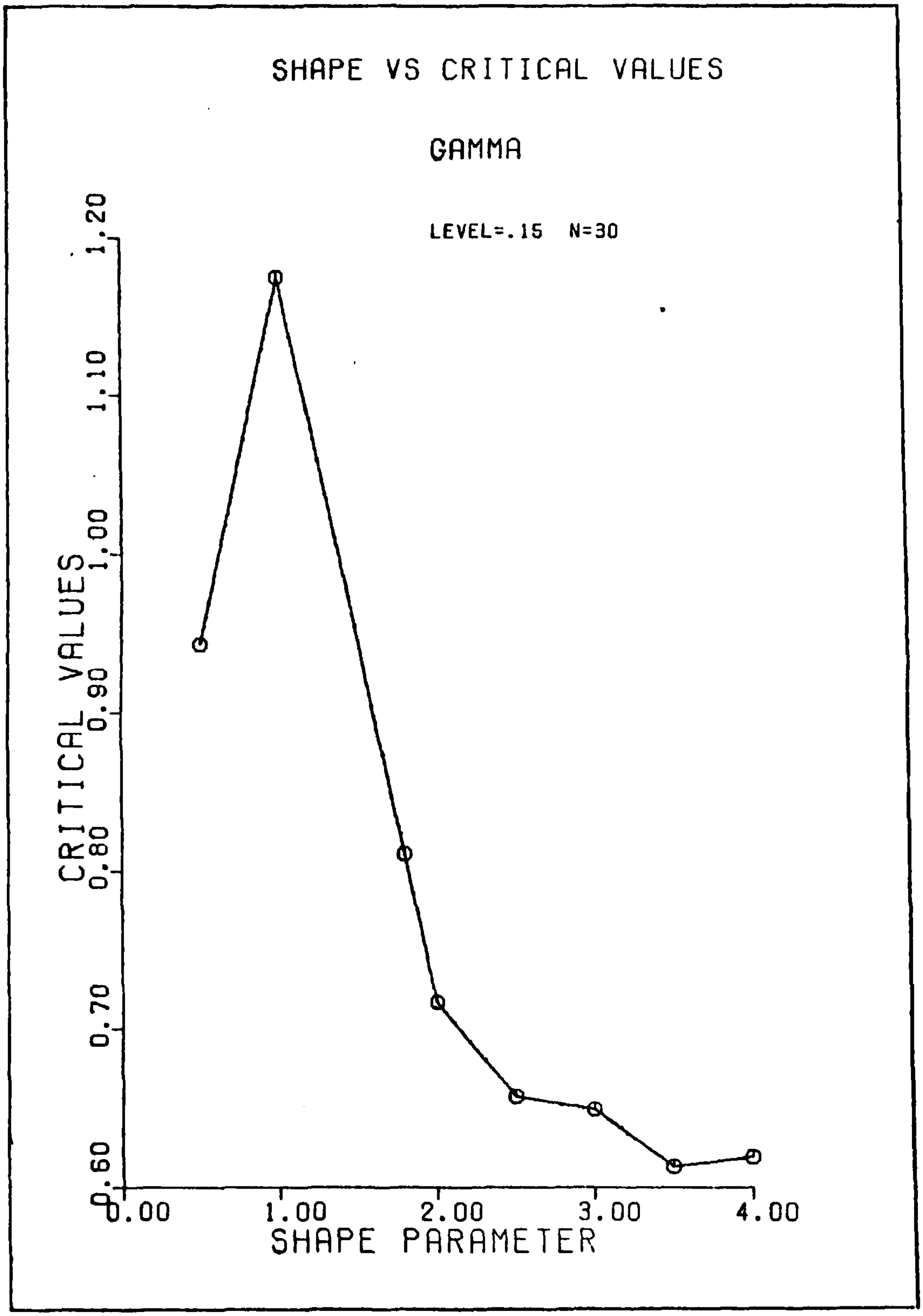


FIG 56. Shape vs A^2 Critical Values, Level = .15, n = 30

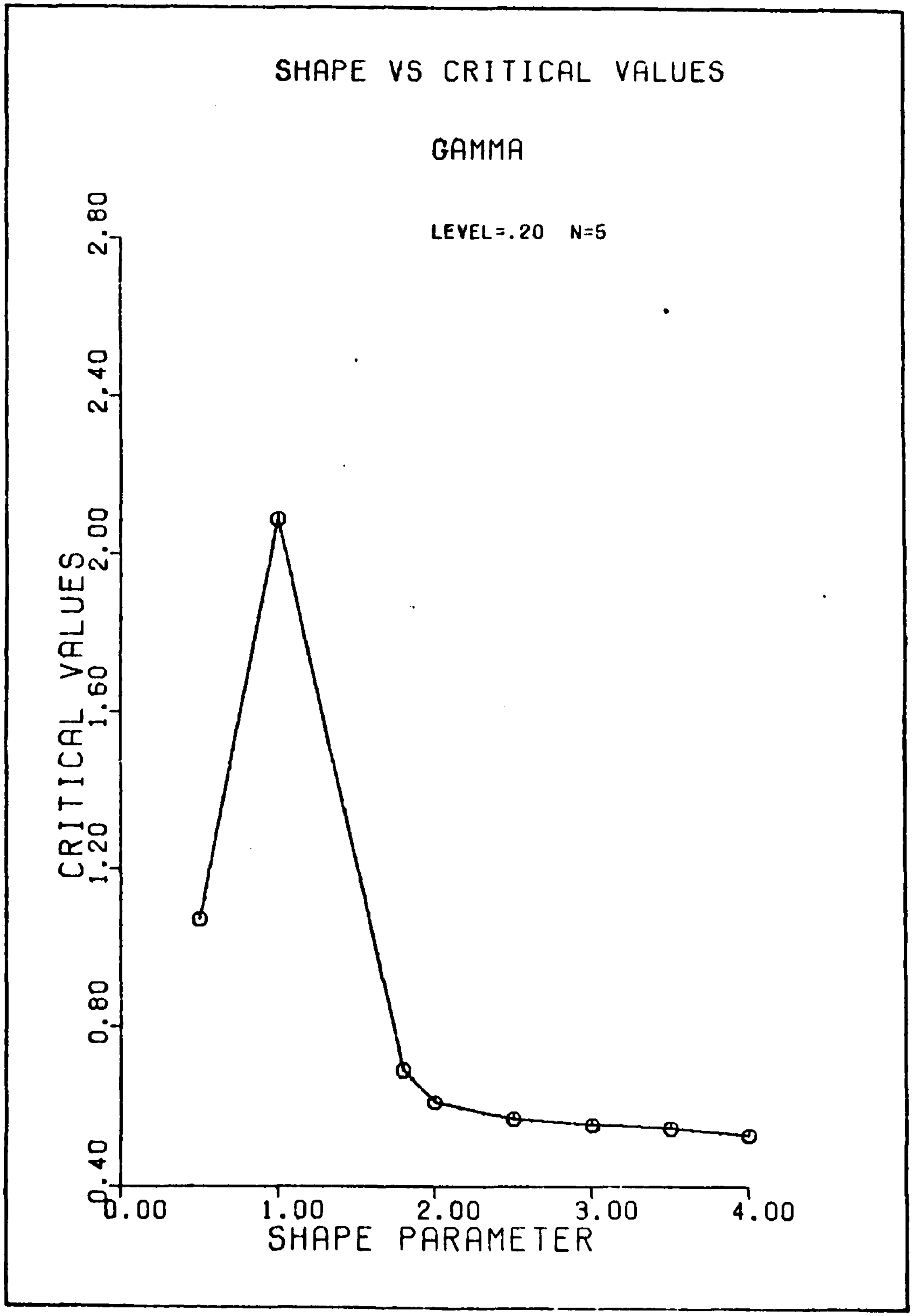


FIG C1. Shape vs A^2 Critical Values, Level = .20, n = 5

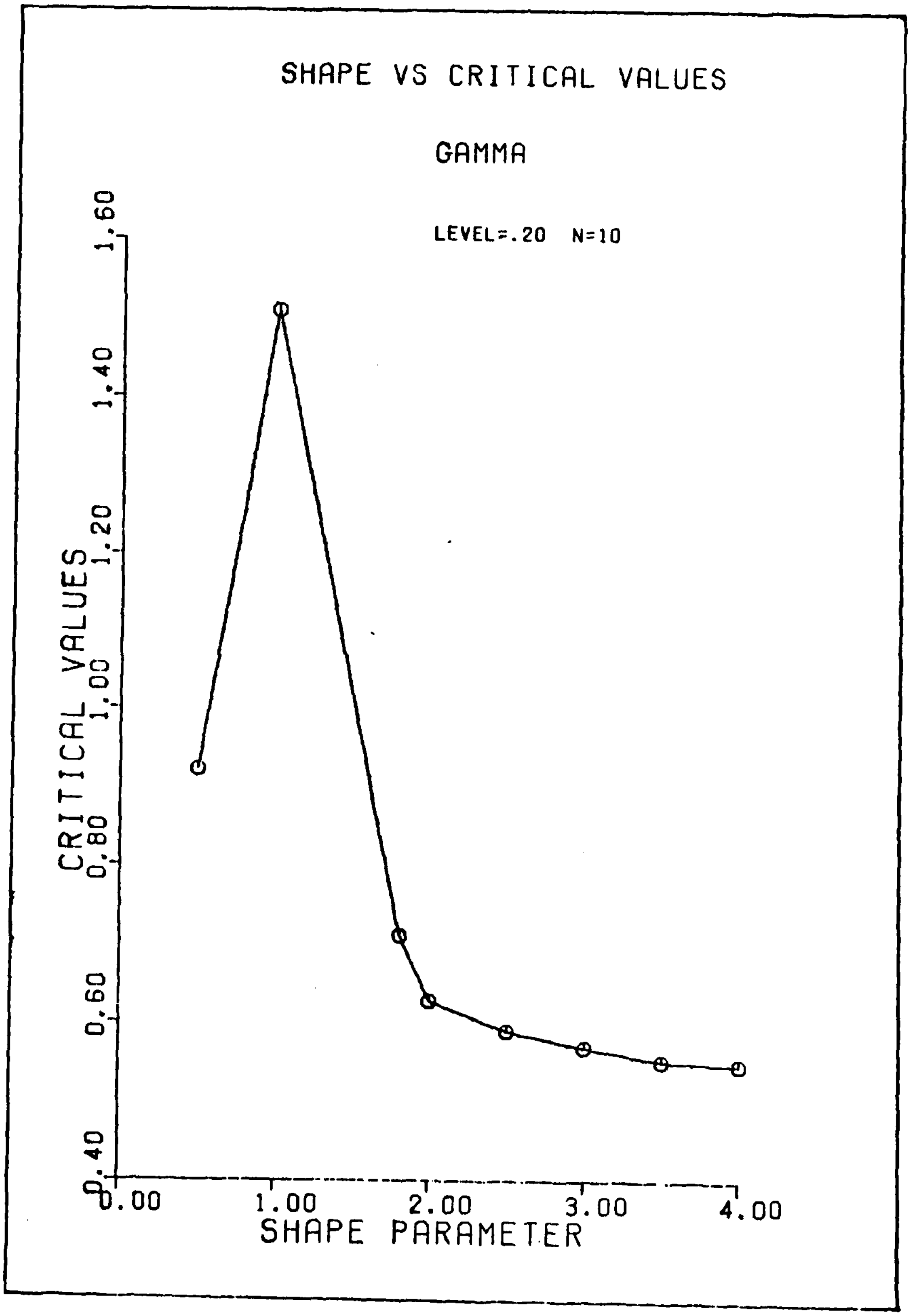


FIG 62. Shape vs A^2 Critical Values, Level = .20, n = 10

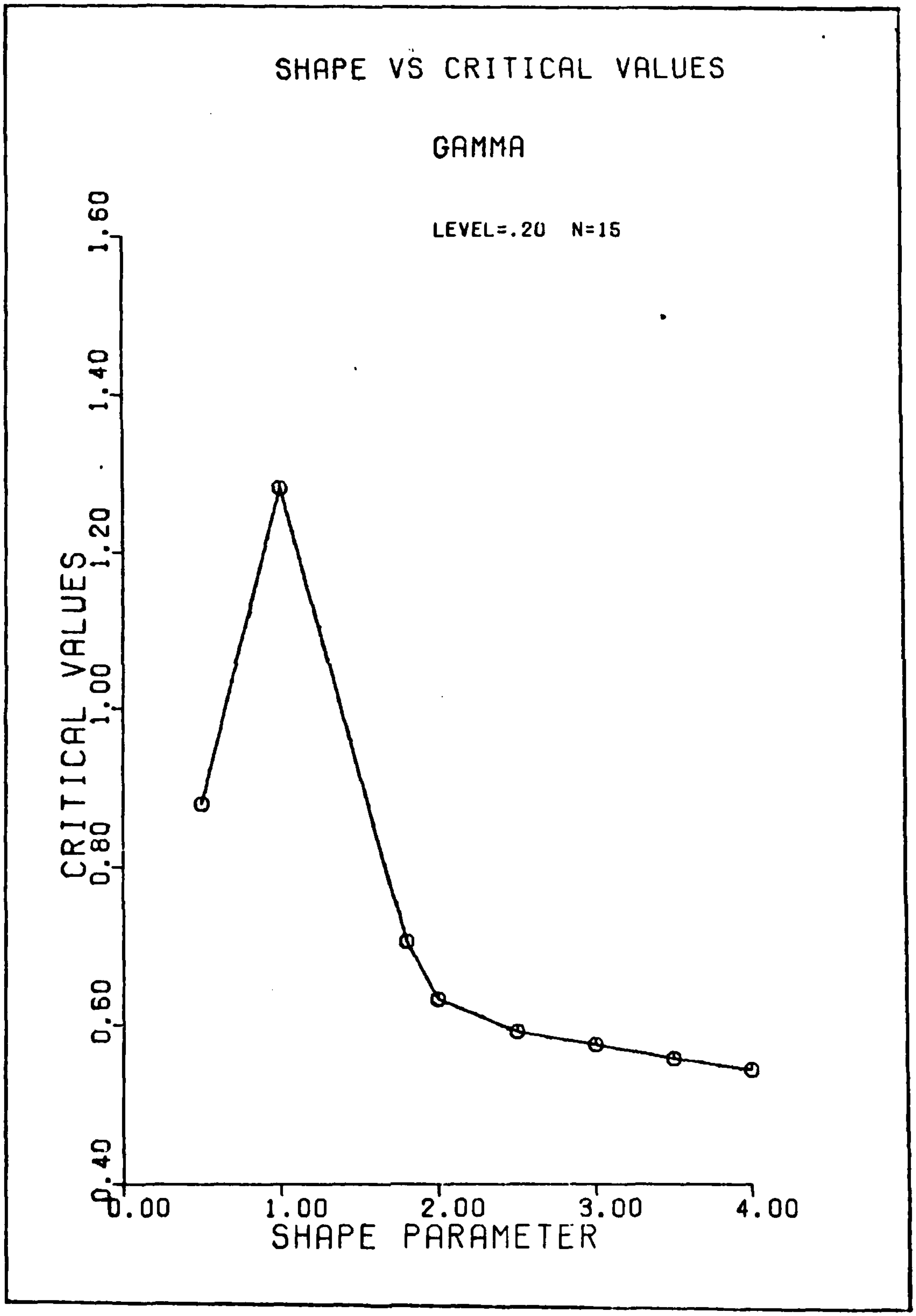


FIG 63. Shape vs A^2 Critical Values, Level = .20, n = 15

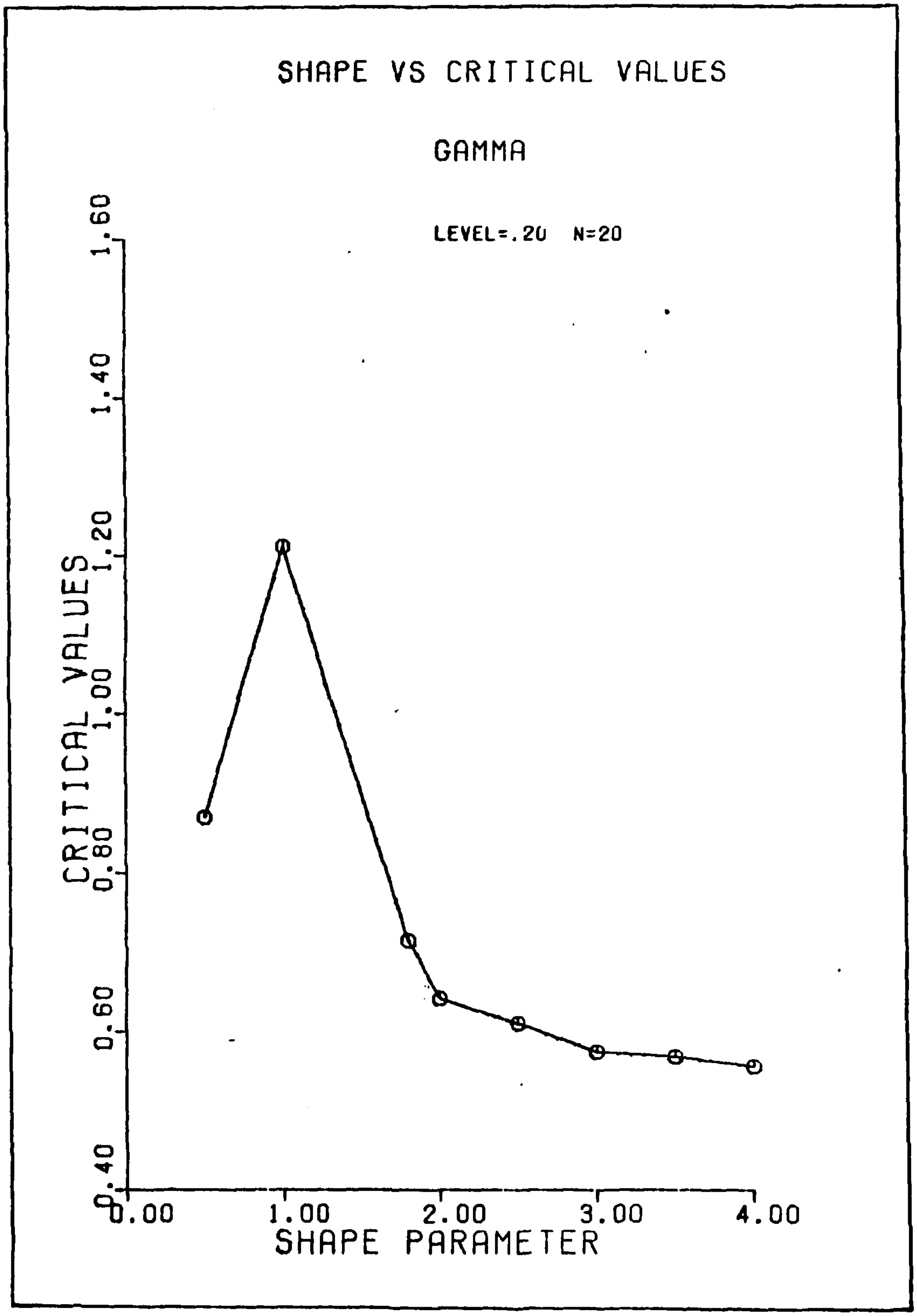


FIG 64. Shape vs A^2 Critical Values, Level = .20, n = 20

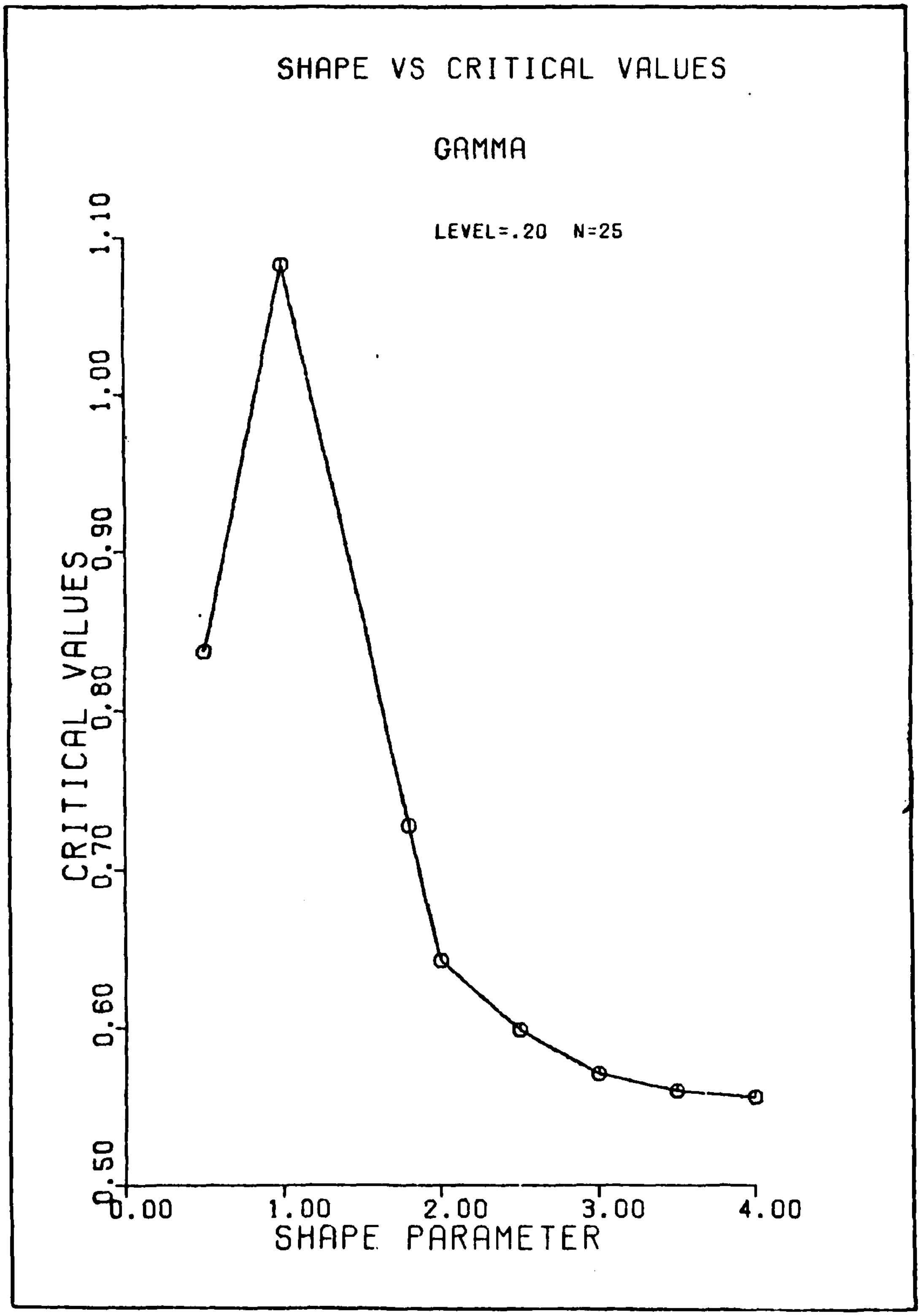


FIG 65. Shape vs A^2 Critical Values, Level = .20, n = 25

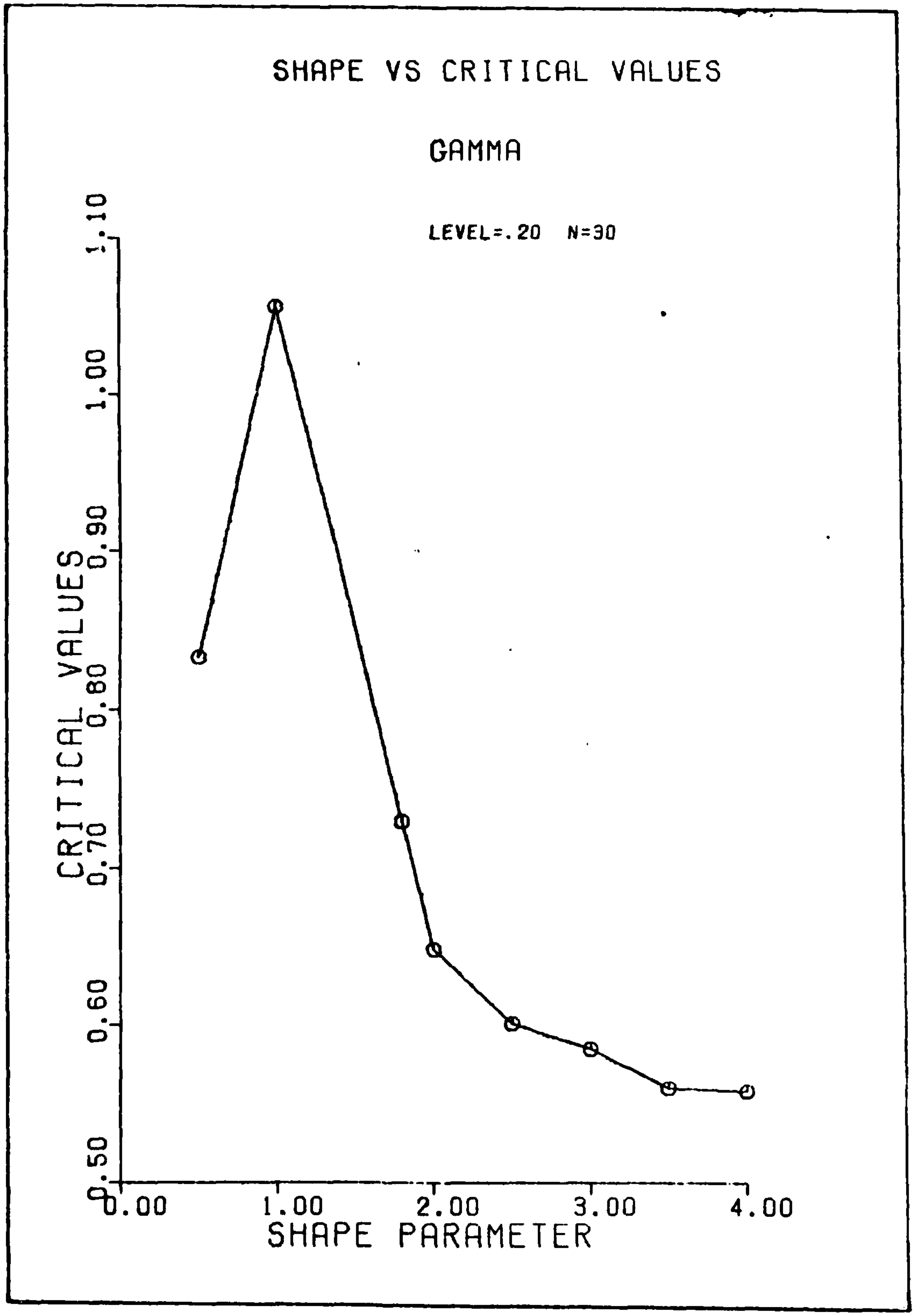


FIG 66. Shape vs A^2 Critical Values, Level = .20, n = 30

APPENDIX F

Graphs of the Cramer-von Mises

Critical Values Verses the

Gamma Shape Parameters

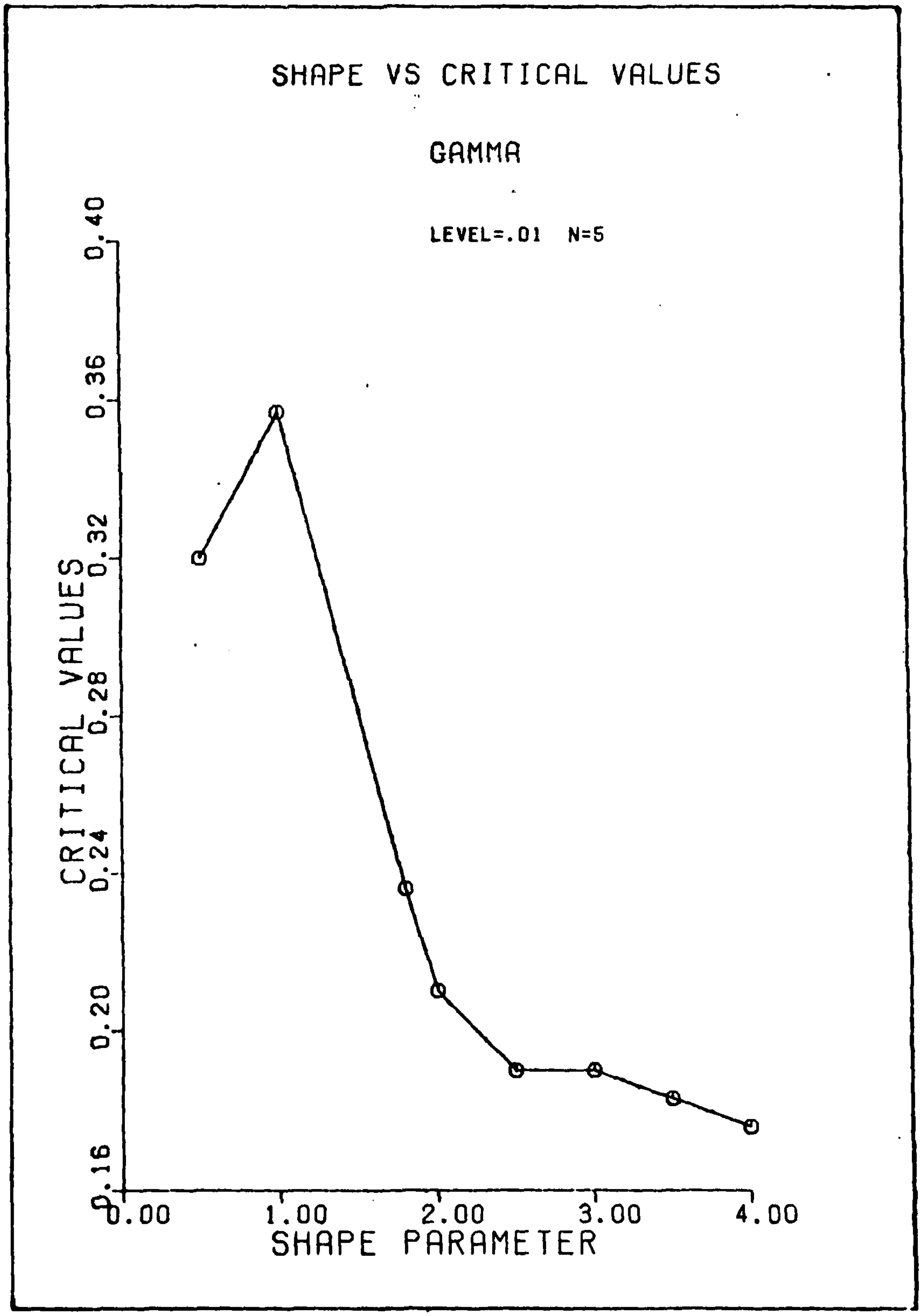


FIG 67. Shape vs W^2 Critical Values, Level = .01, n = 5

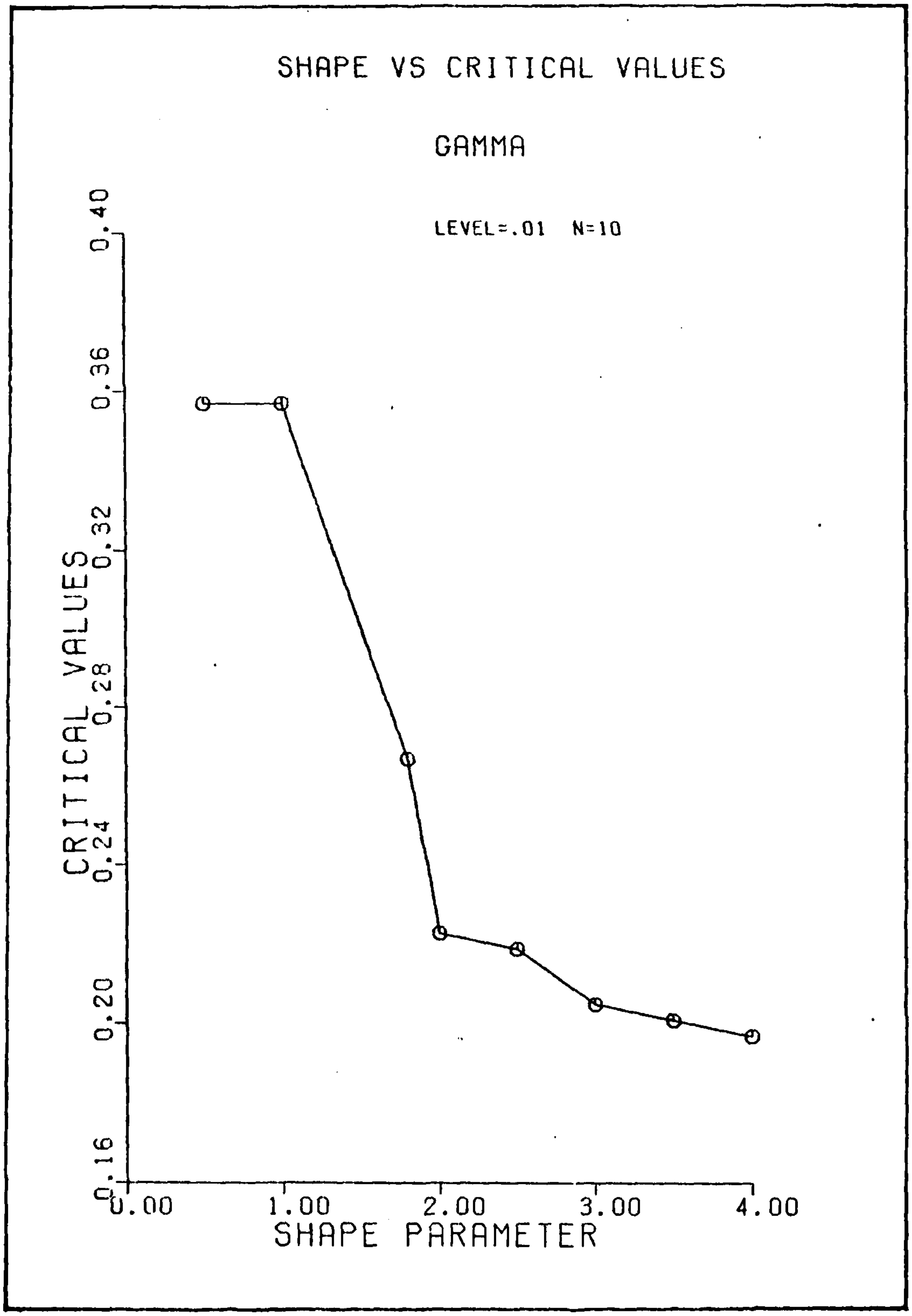


FIG 68. Shape vs W^2 Critical Values, Level = .01, n = 10

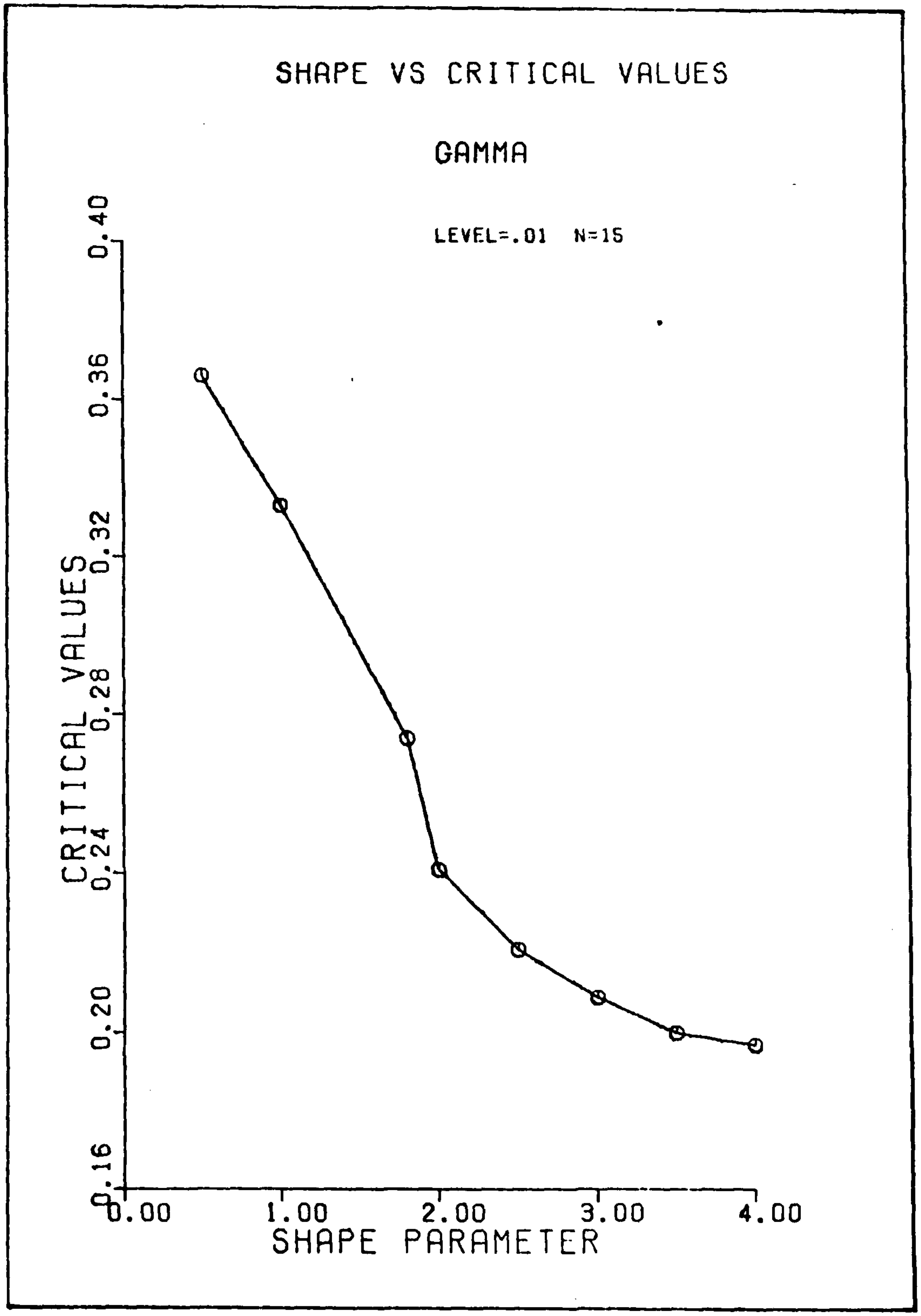


FIG 69. Shape vs [illegible]^2 Critical Values, Level = .01, n = 15

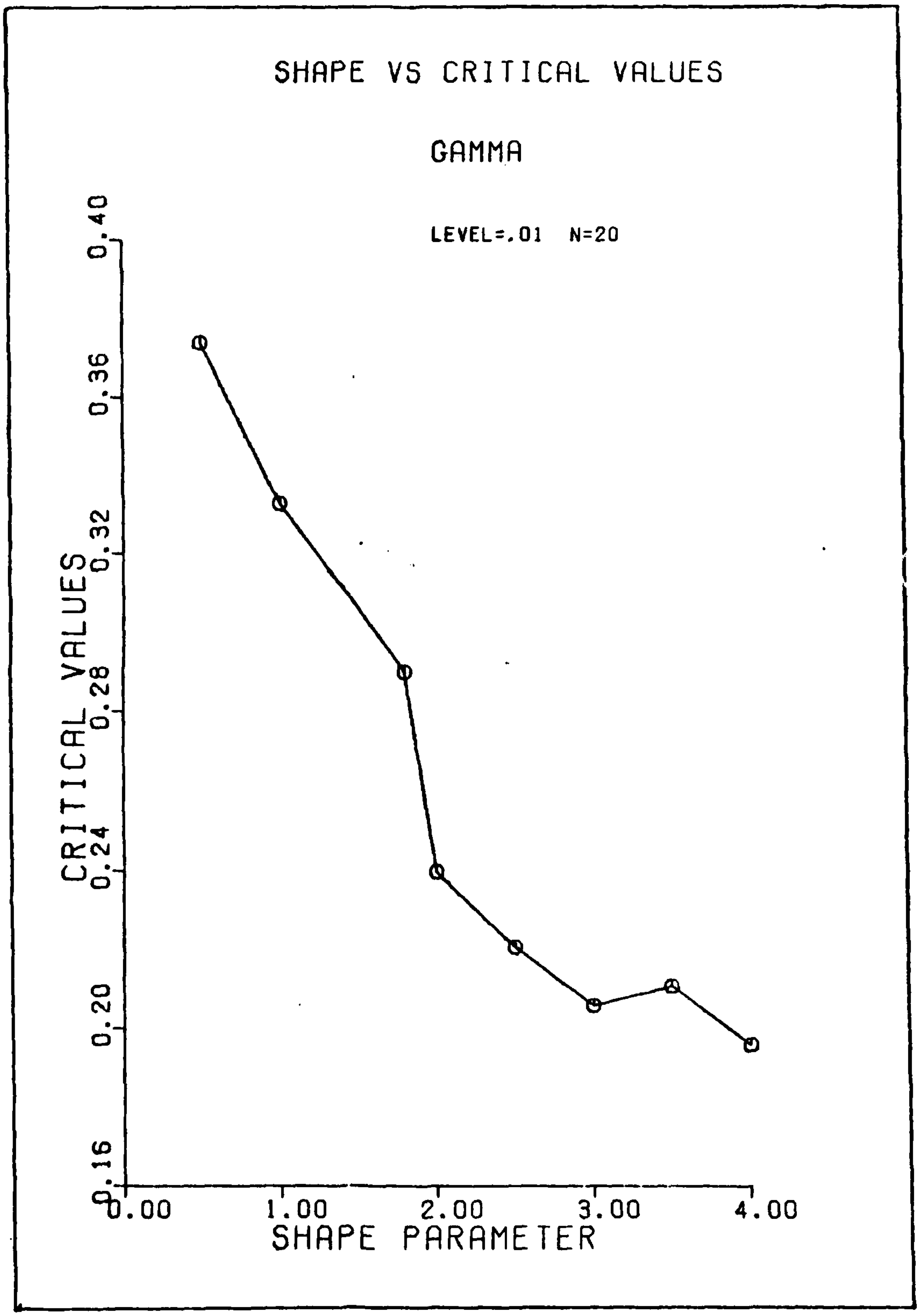


FIG 7D. Shape vs W^2 Critical Values, Level = .01, n = 20

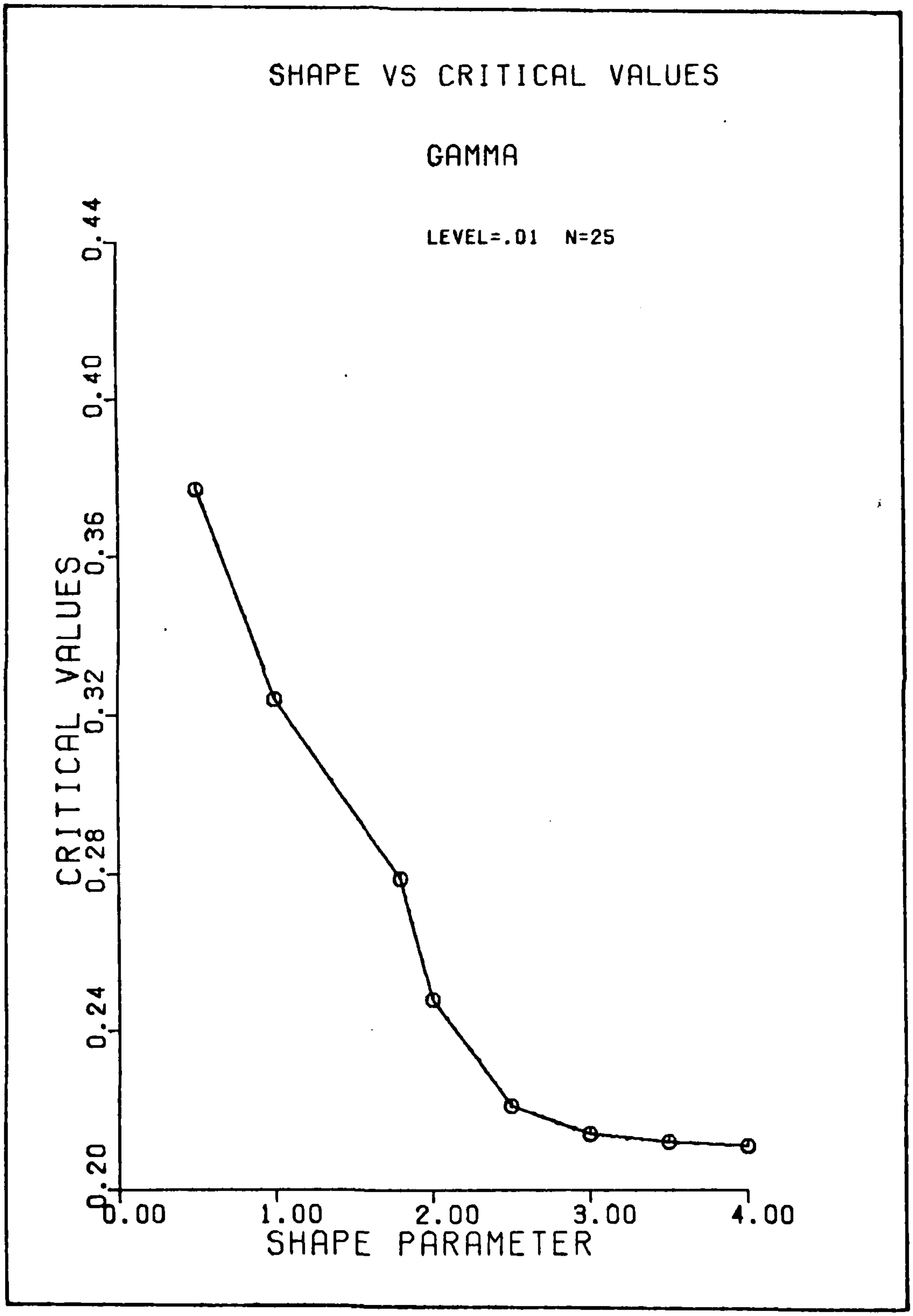


FIG 71. Shape vs W^2 Critical Values, Level = .01, n = 25

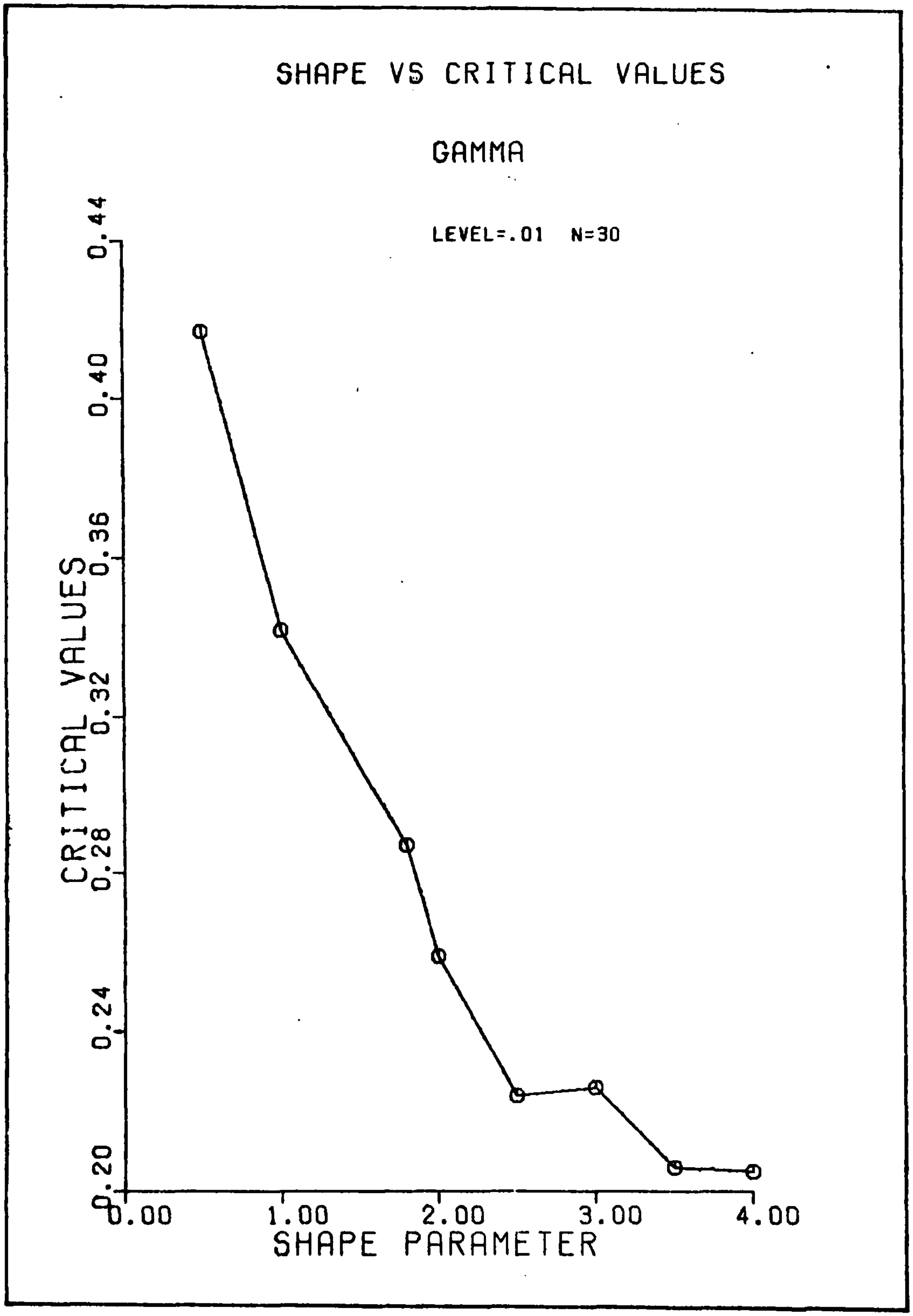


FIG 72. Shape vs W^2 Critical Values, Level = .01, n = 30

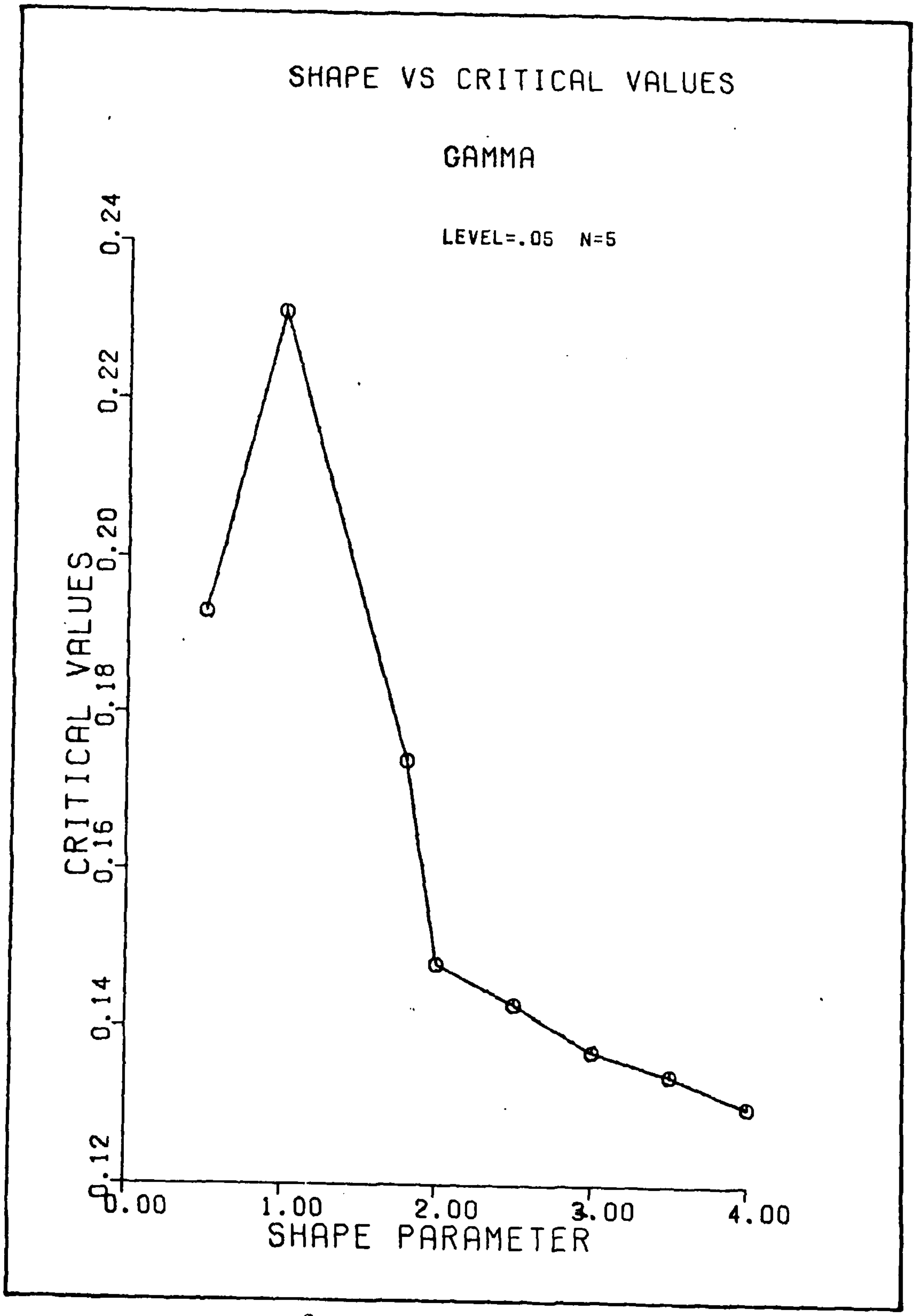


FIG 73. Shape vs χ^2 Critical Values, Level = .05, n = 5

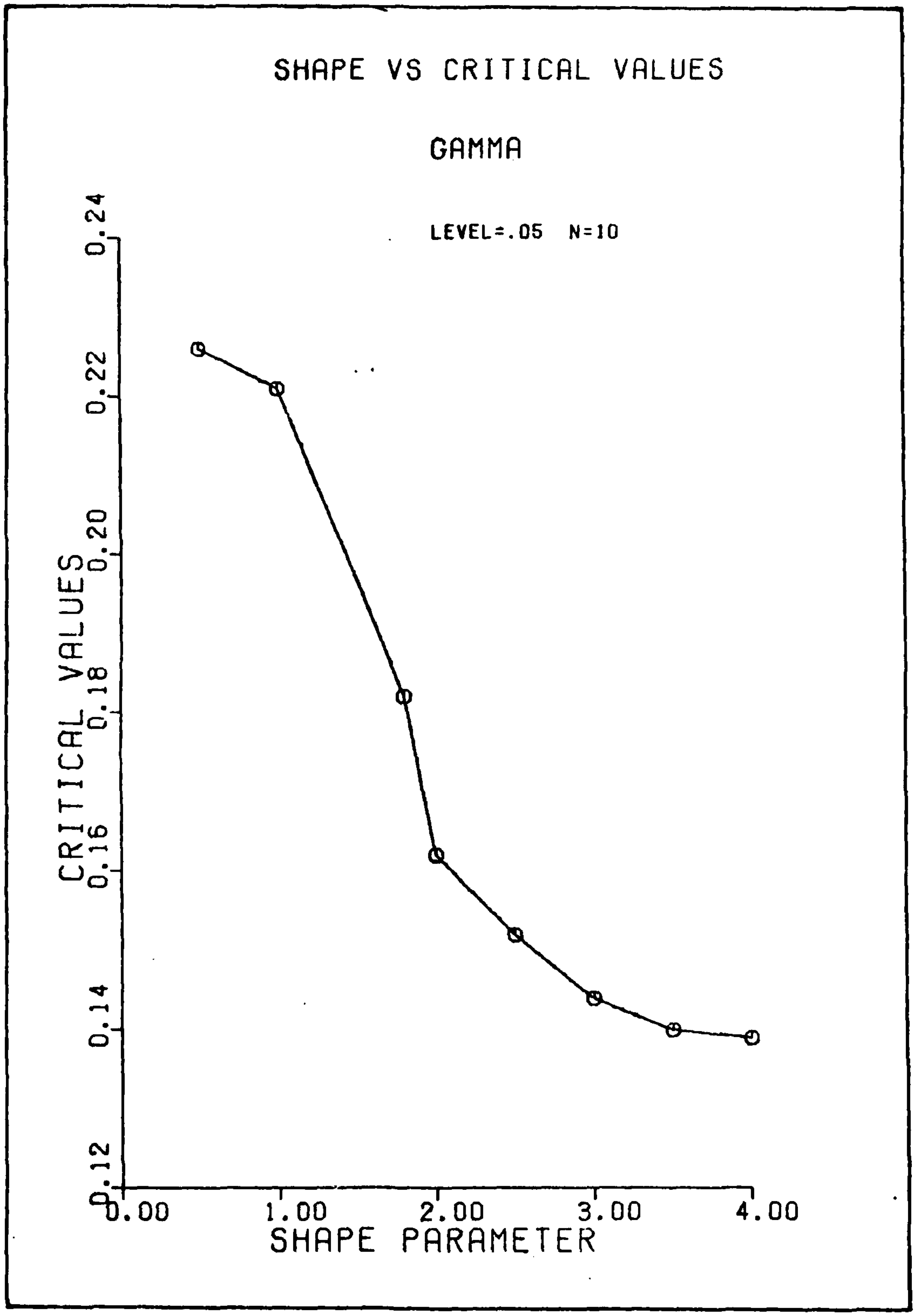


Fig. 74. Shape vs W^2 Critical Values, Level = .05, n = 10

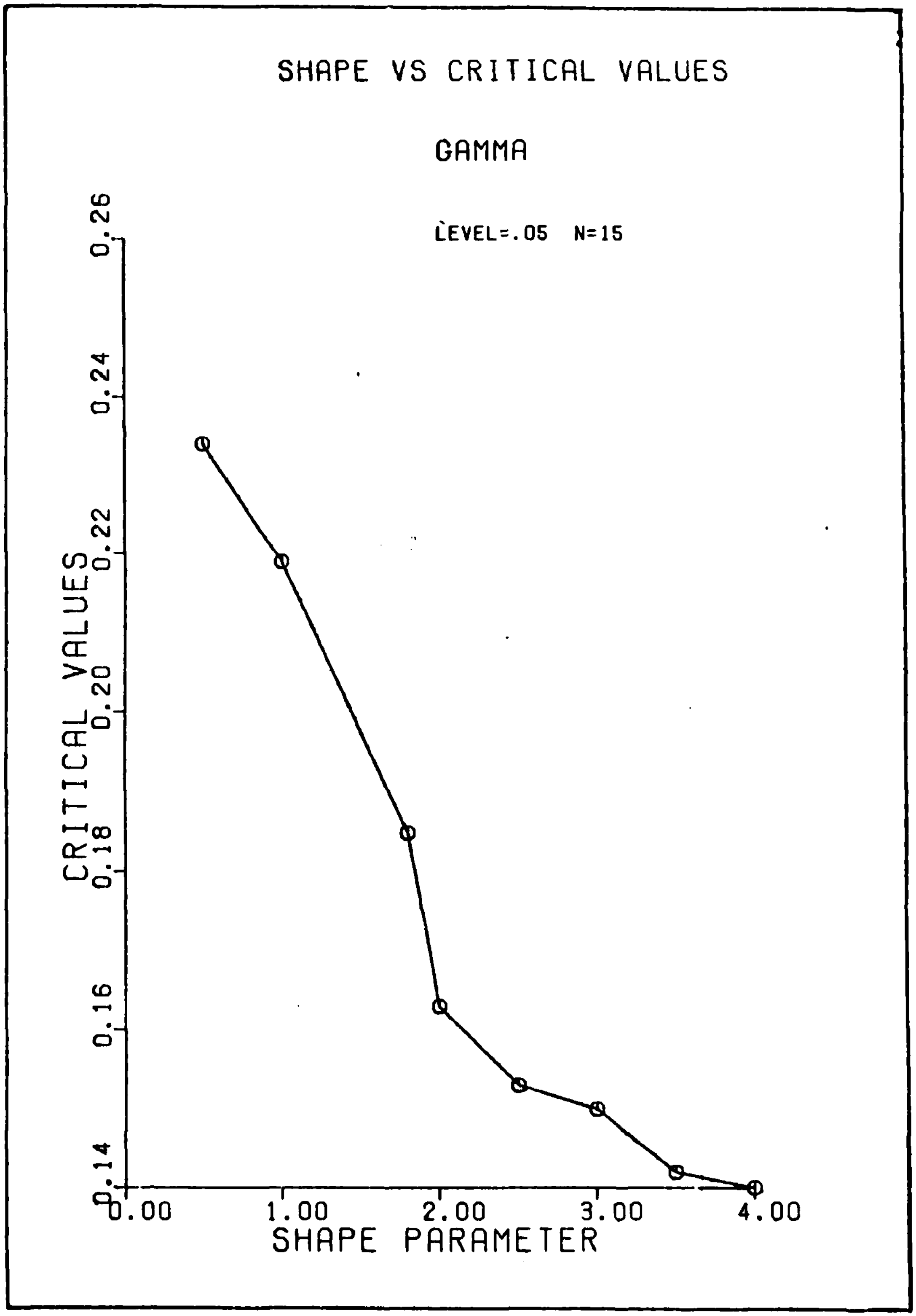


FIG 75. Shape vs W^2 Critical Values, Level = .05, n = 15

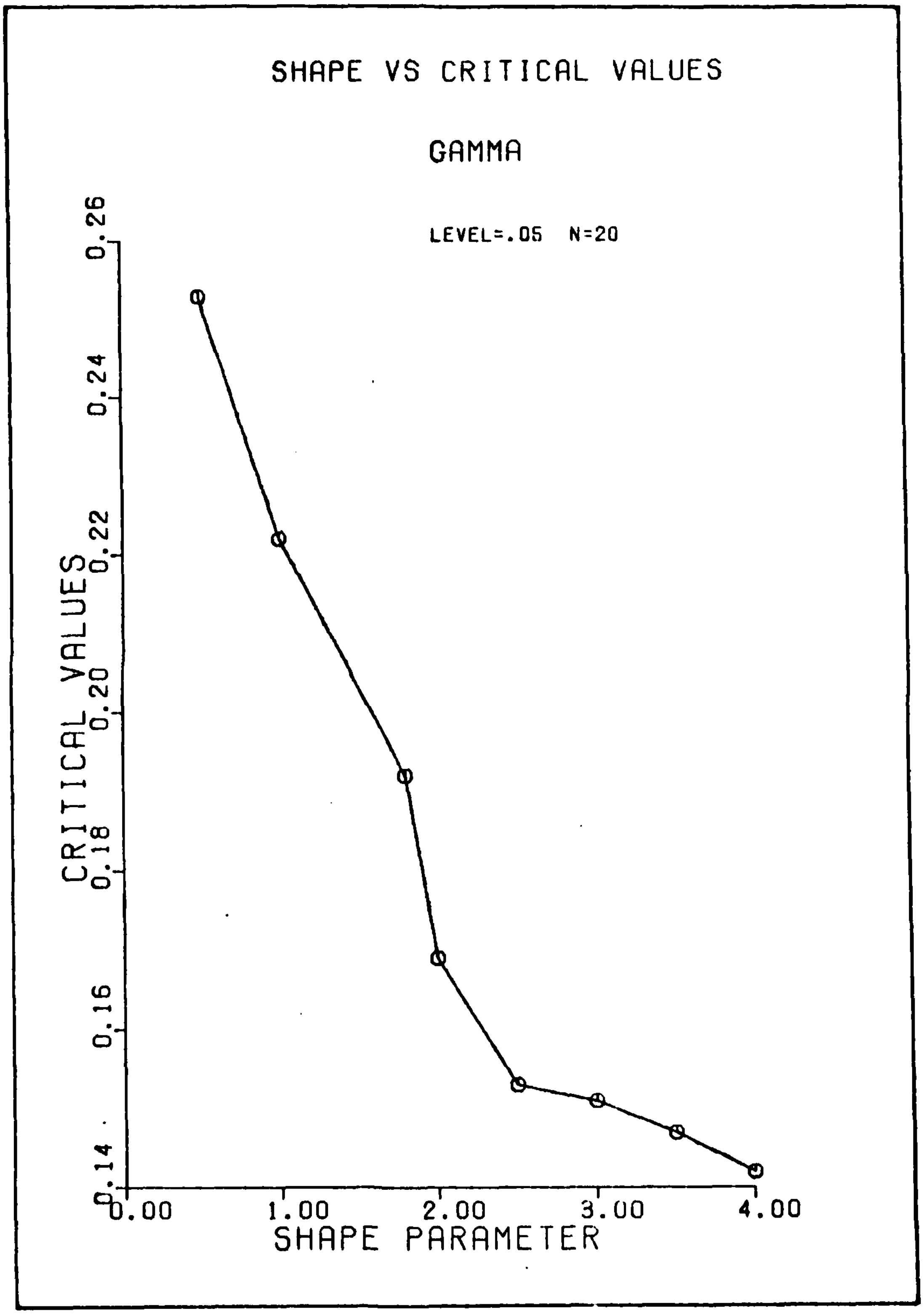


FIG 73. Shape vs W^2 Critical Values, Level = .05, n = 20

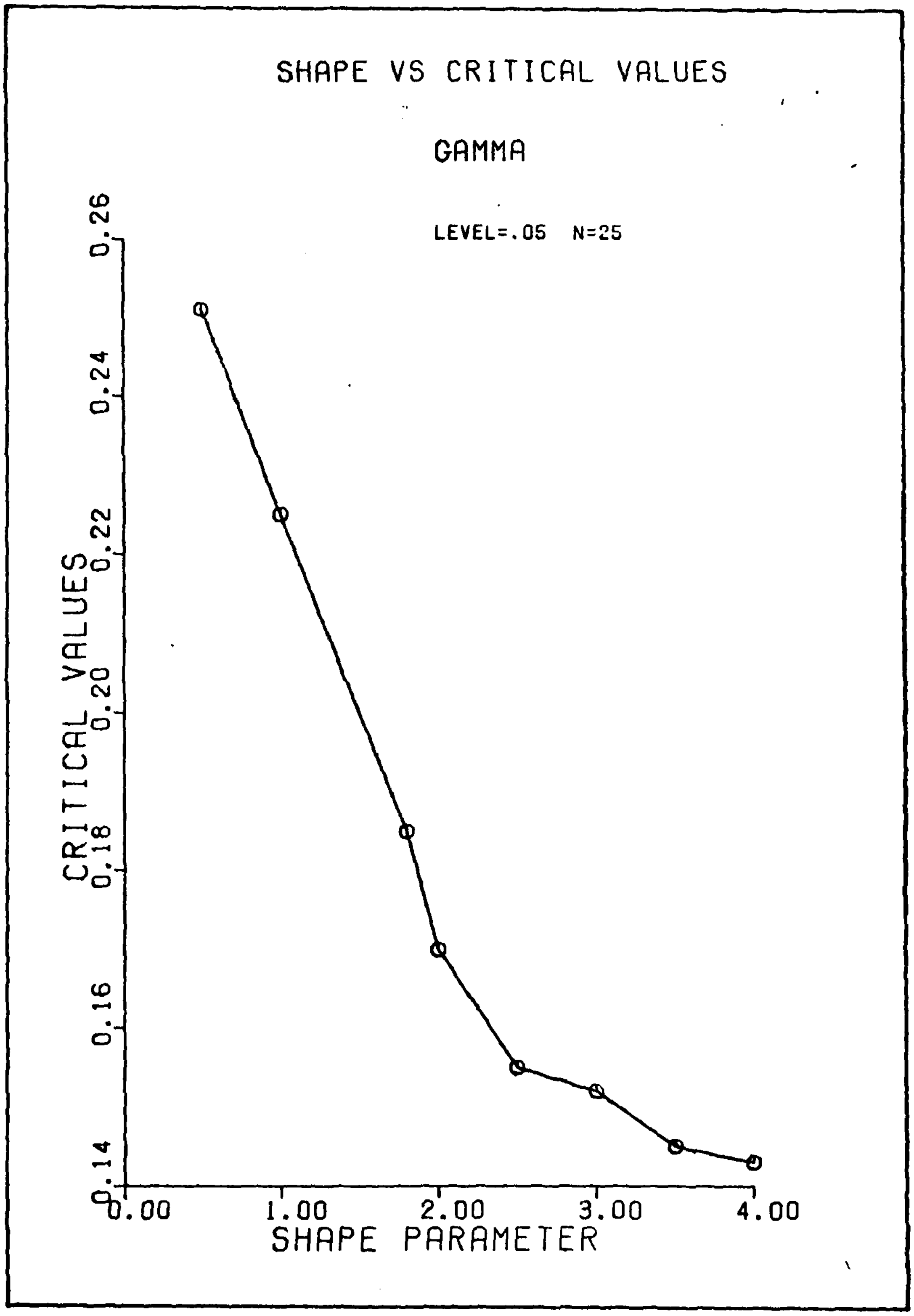


FIG 77. Shape vs W^2 Critical Values, Level = .05, n = 25

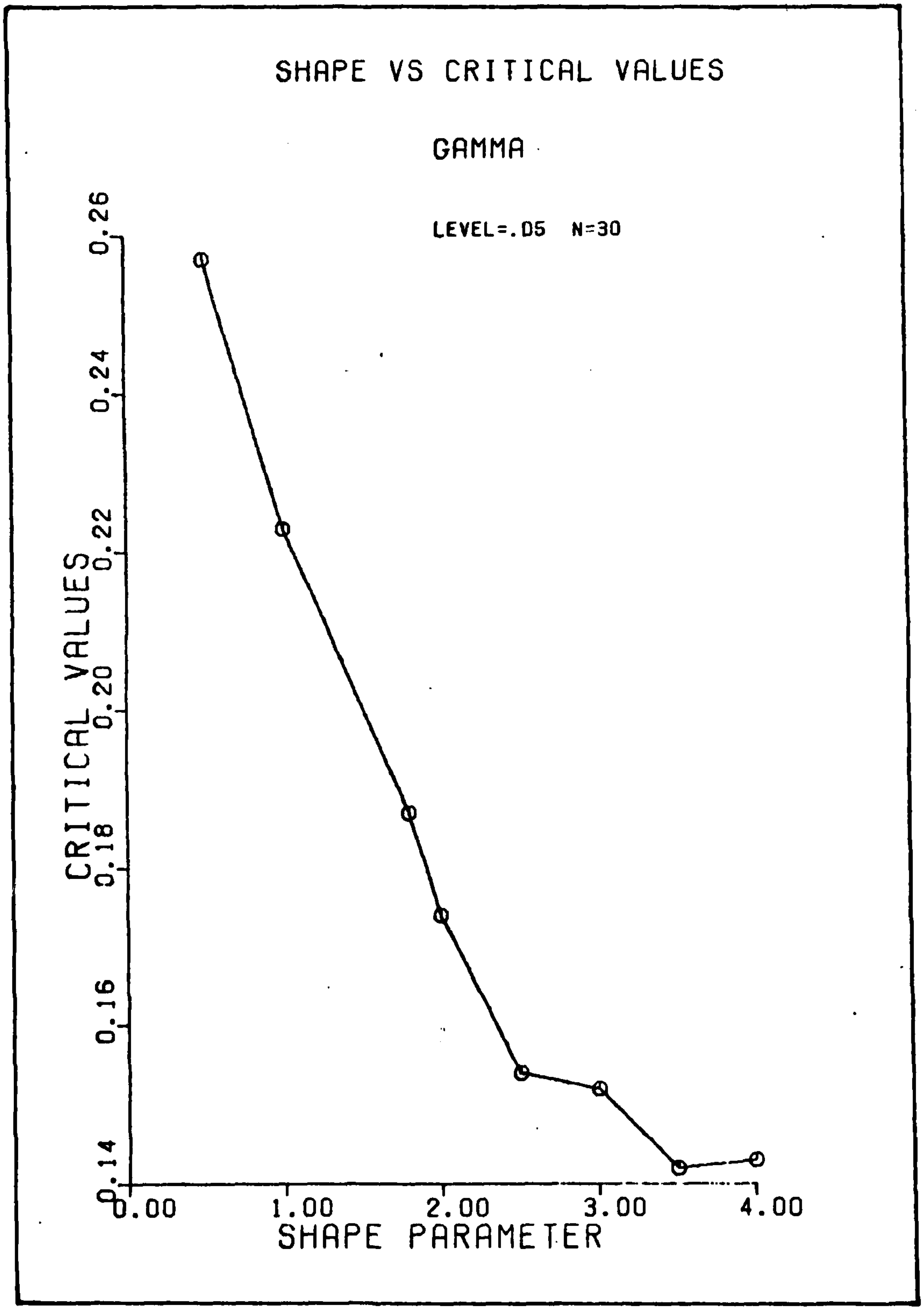


FIG 78. Shape vs W^2 Critical Values, Level = .05, n = 30

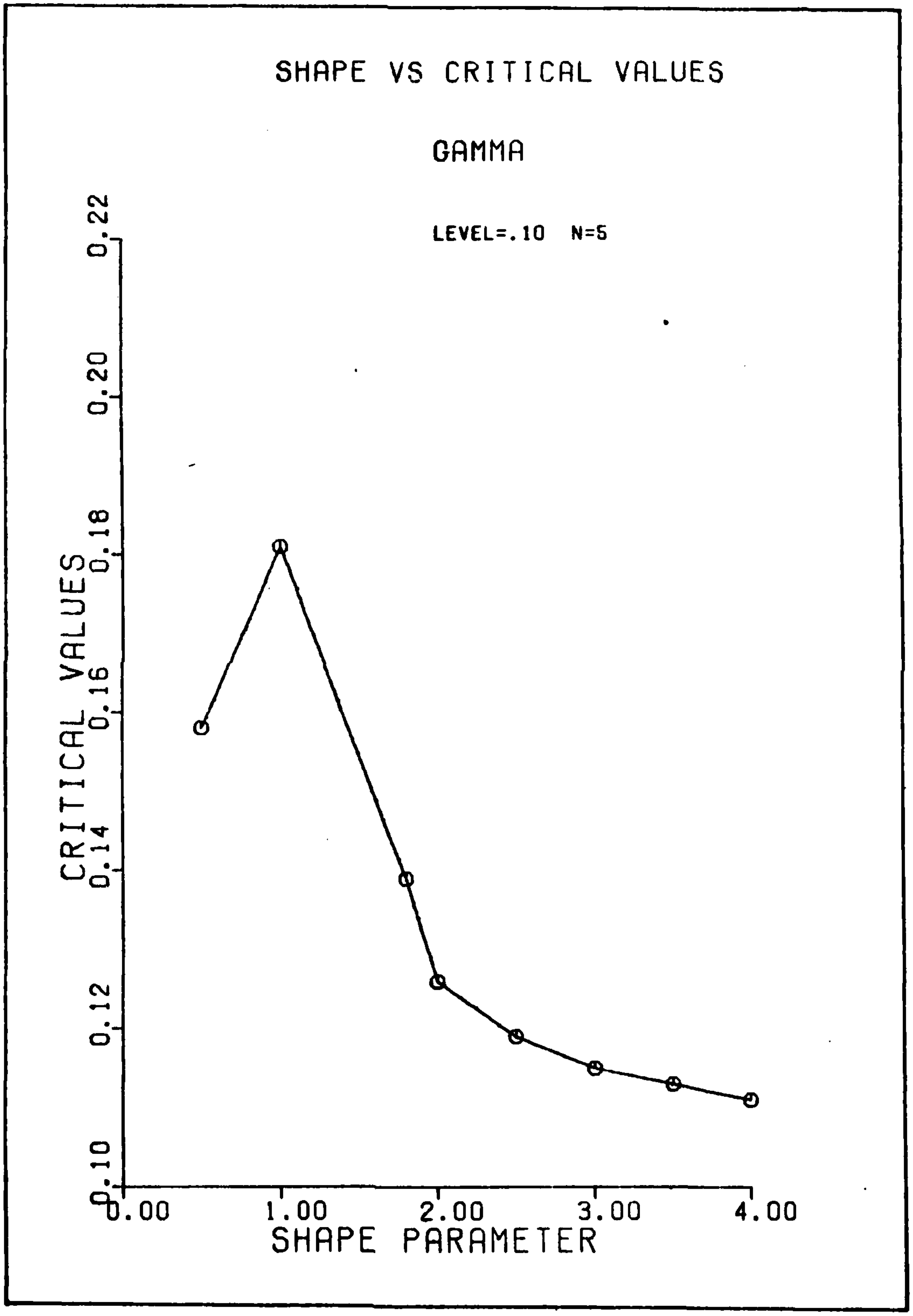


FIG 79. Shape vs W^2 Critical Values, Level = .10, n = 5

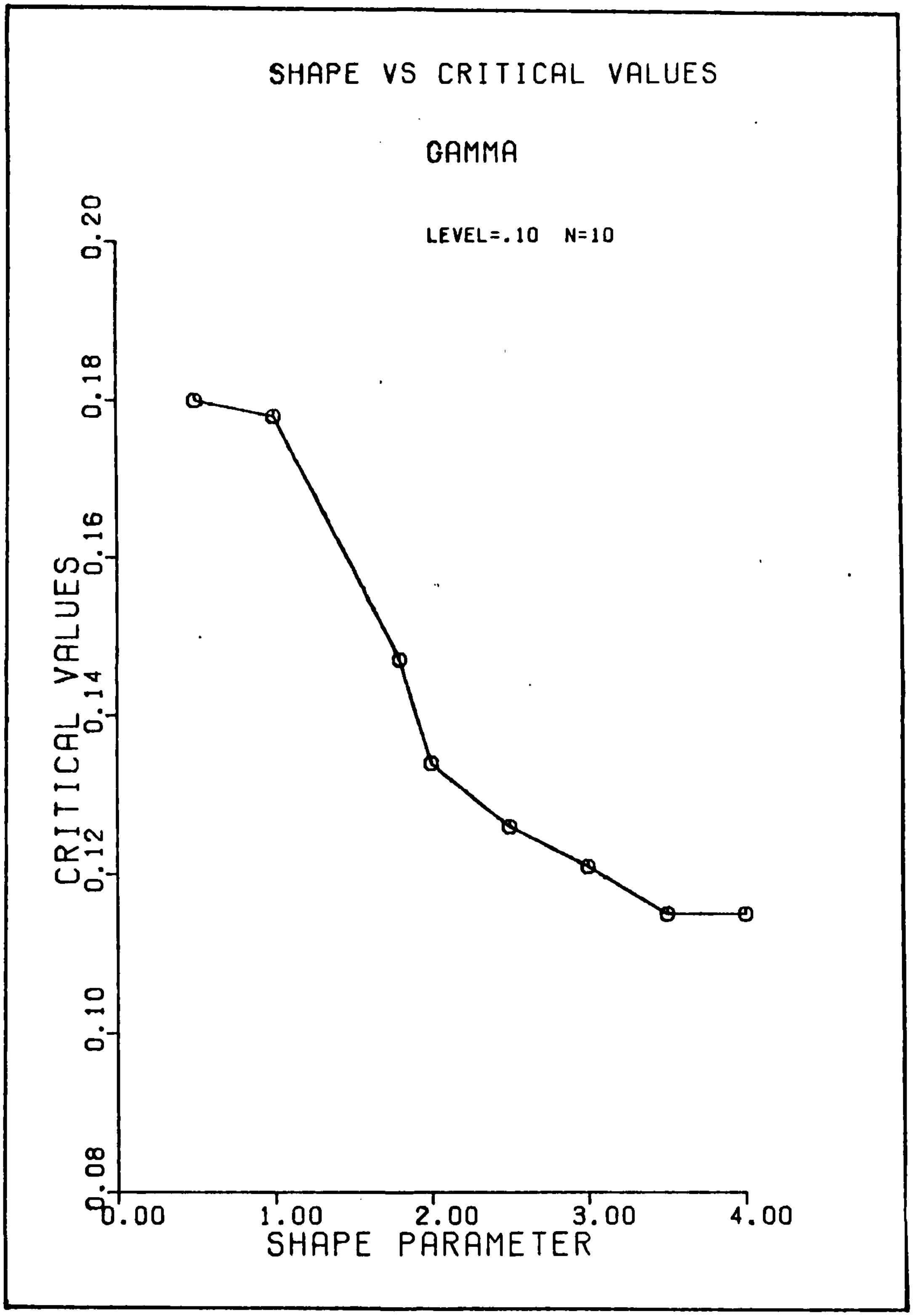


FIG 80. Shape vs W^2 Critical Values, Level = .10, n = 10

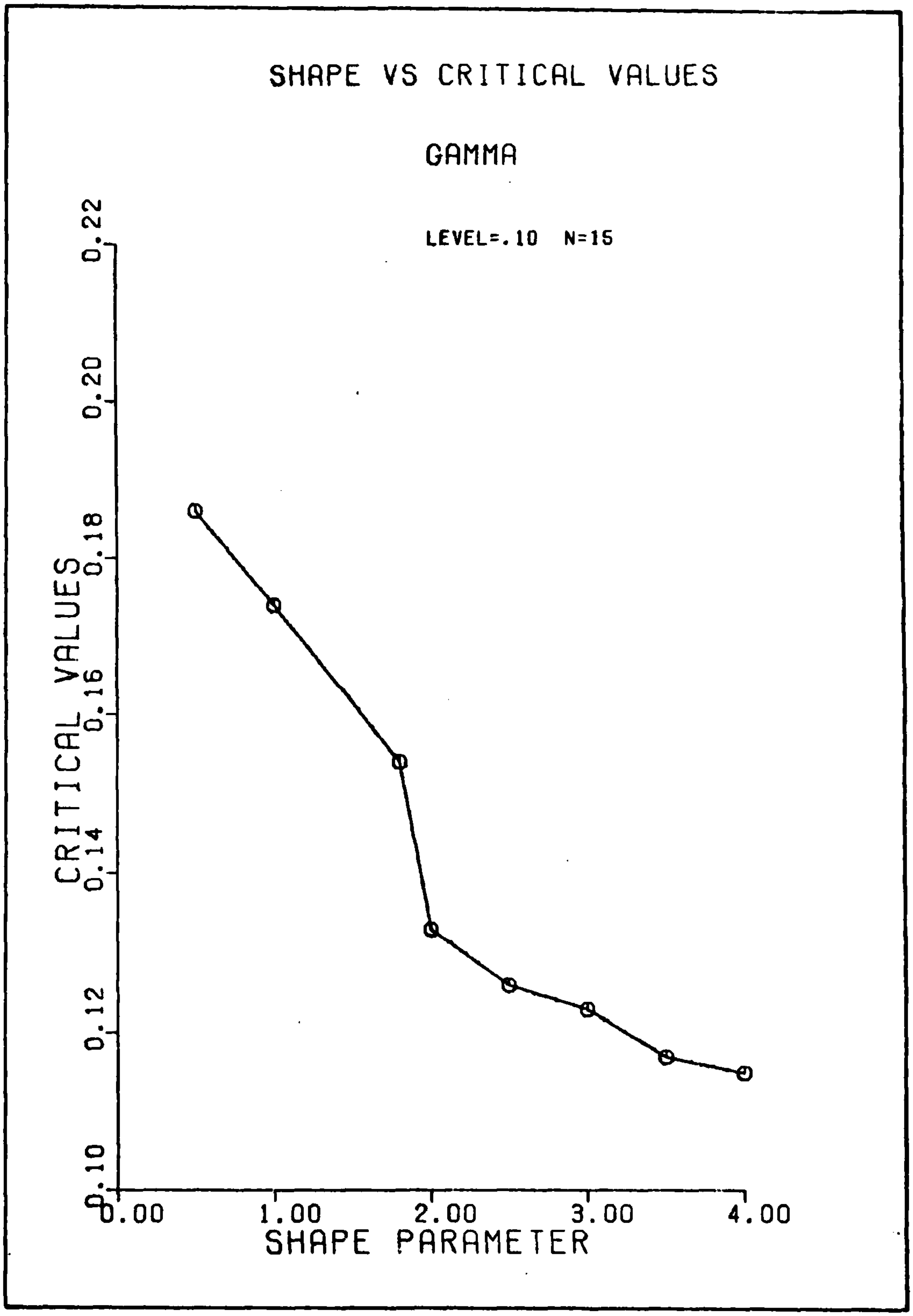


FIG 81. Shape vs W^2 Critical Values, Level = .10, n = 15

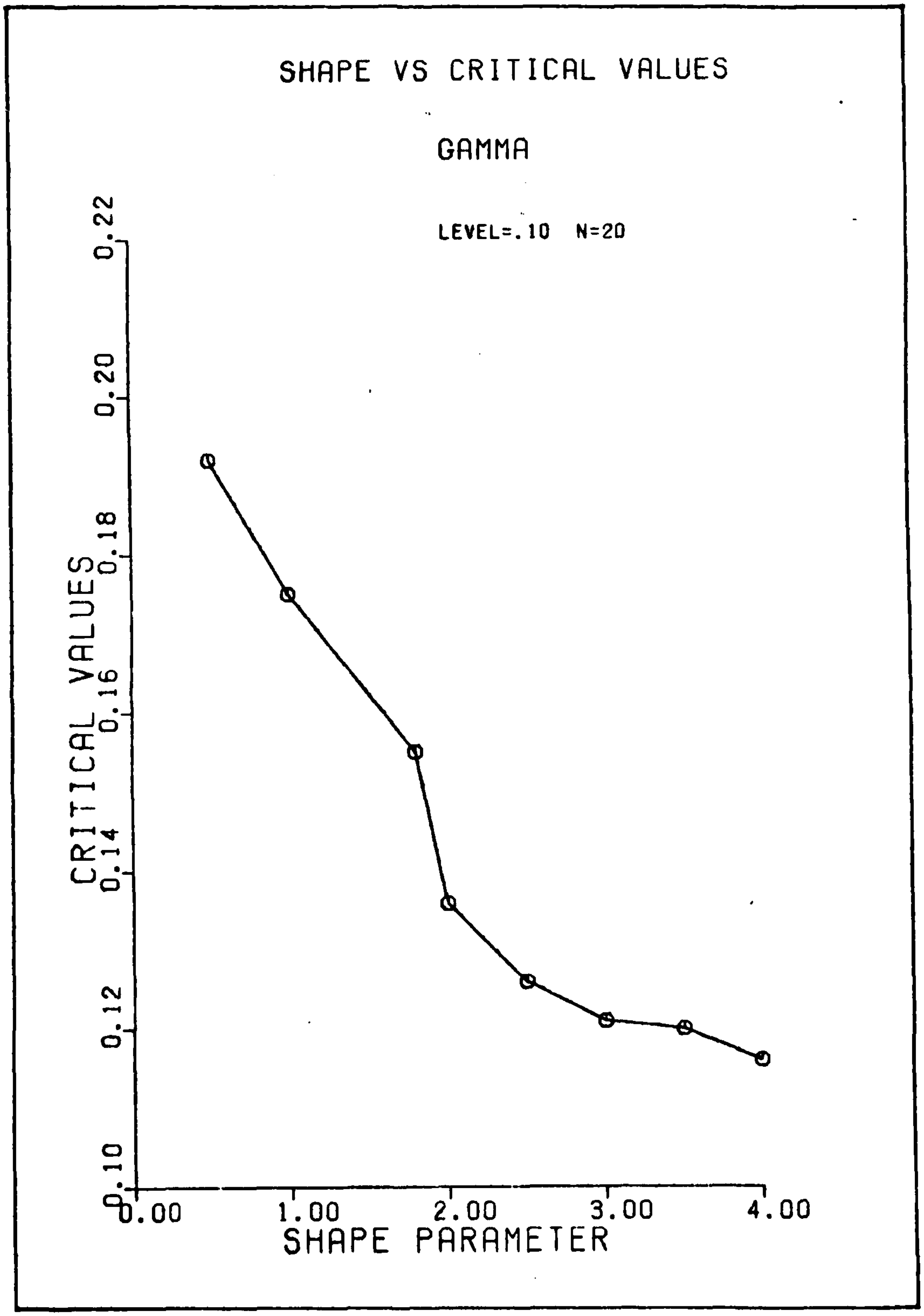


FIG 82. Shape vs W^2 Critical Values, Level = .10, n = 20

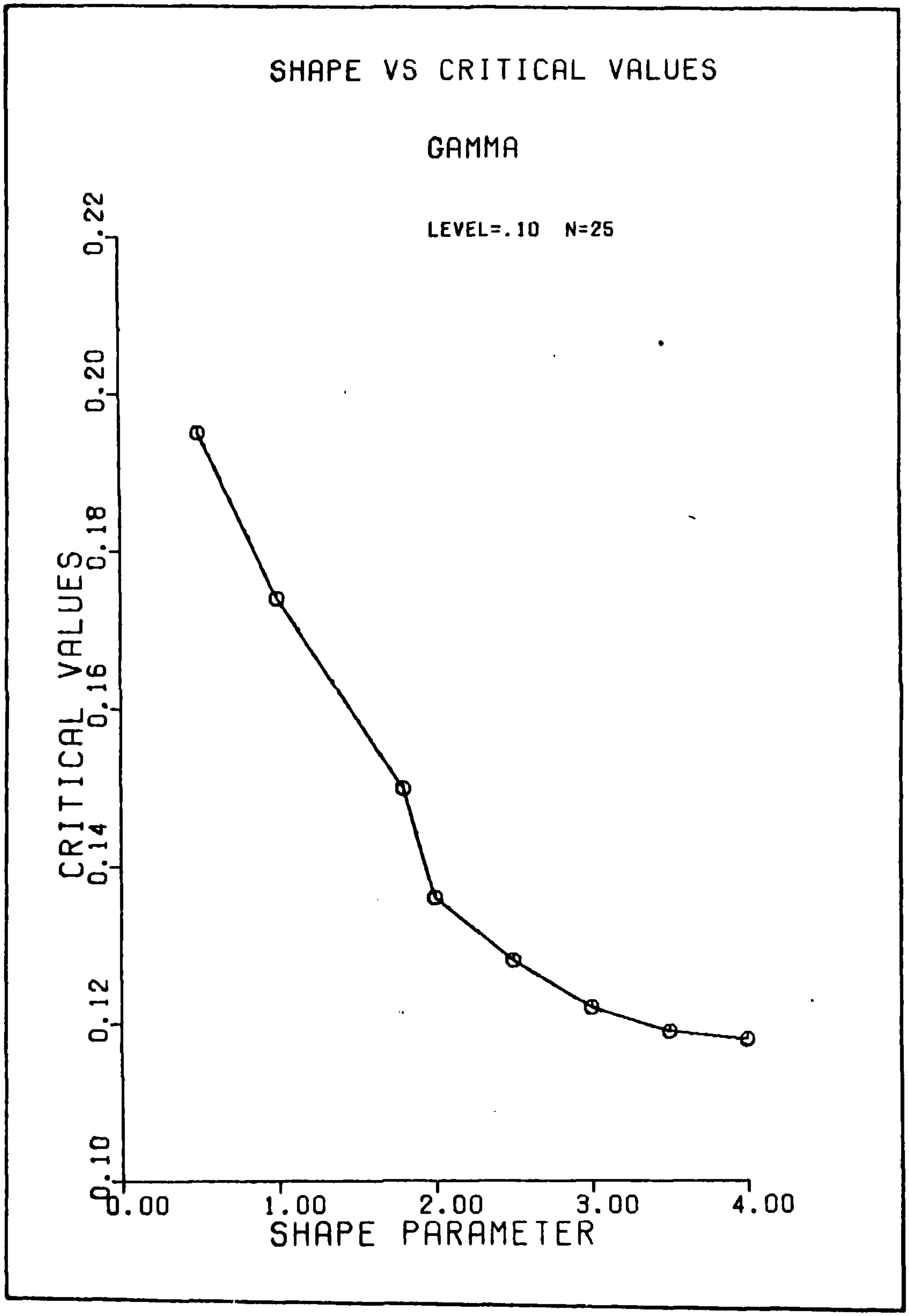


FIG 83. Shape vs W^2 Critical Values, Level = .10, n = 25

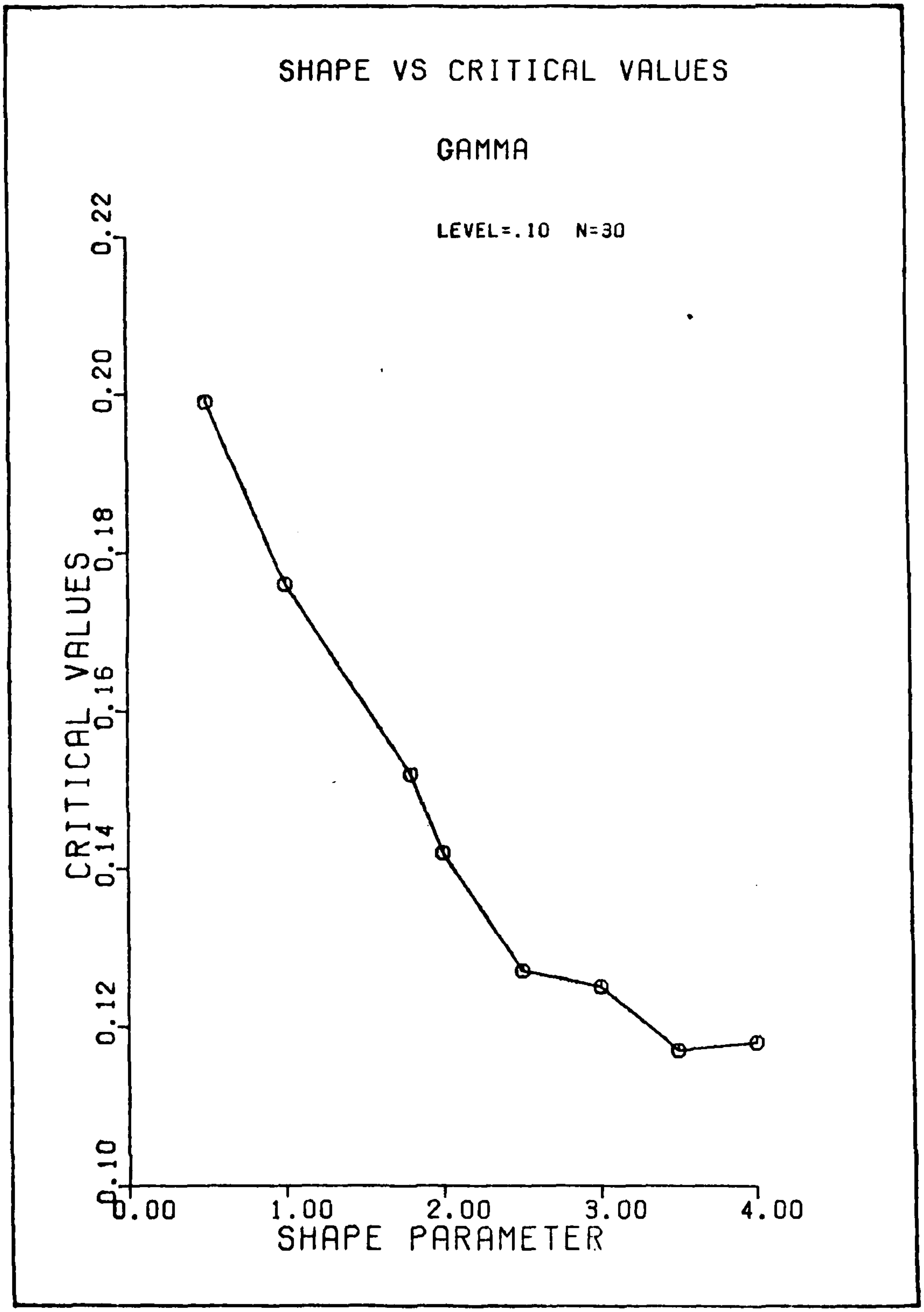


FIG 84. Shape vs W^2 Critical Values, Level = .10, n = 30

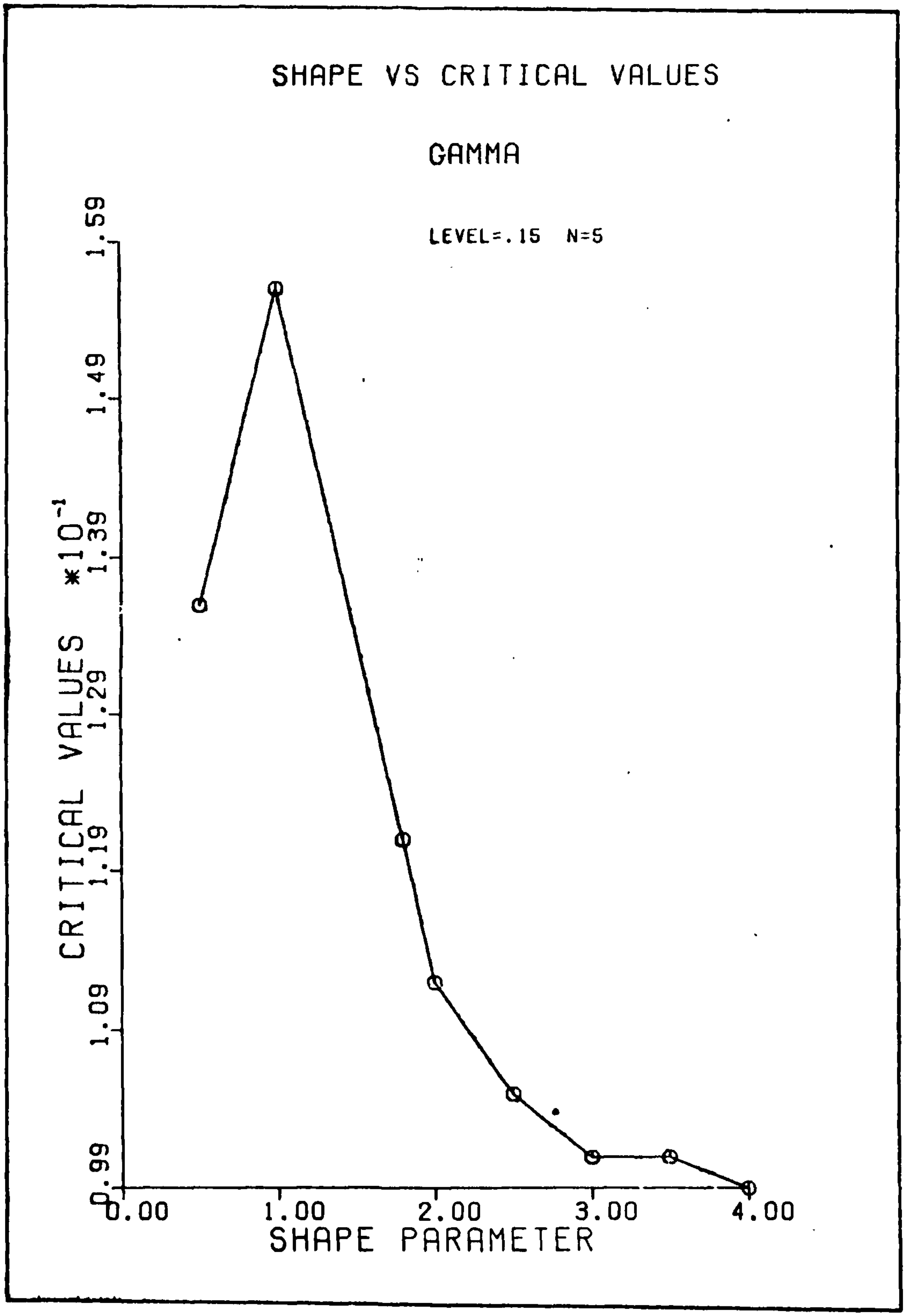


FIG 85. Shape vs W^2 Critical Values, Level = .15, n = 5

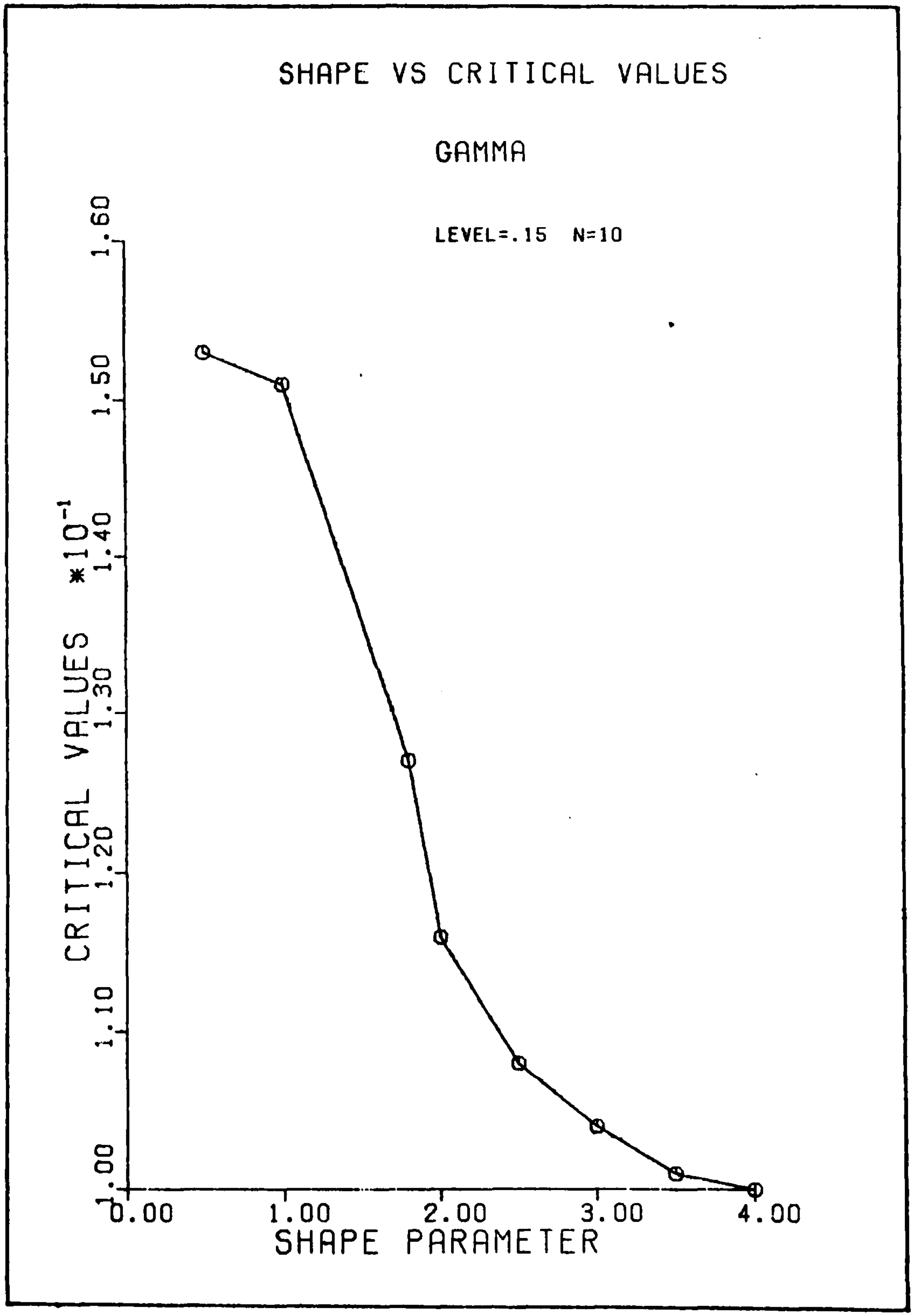


FIG 36. Shape vs W^2 Critical Values, Level = .15, n = 10

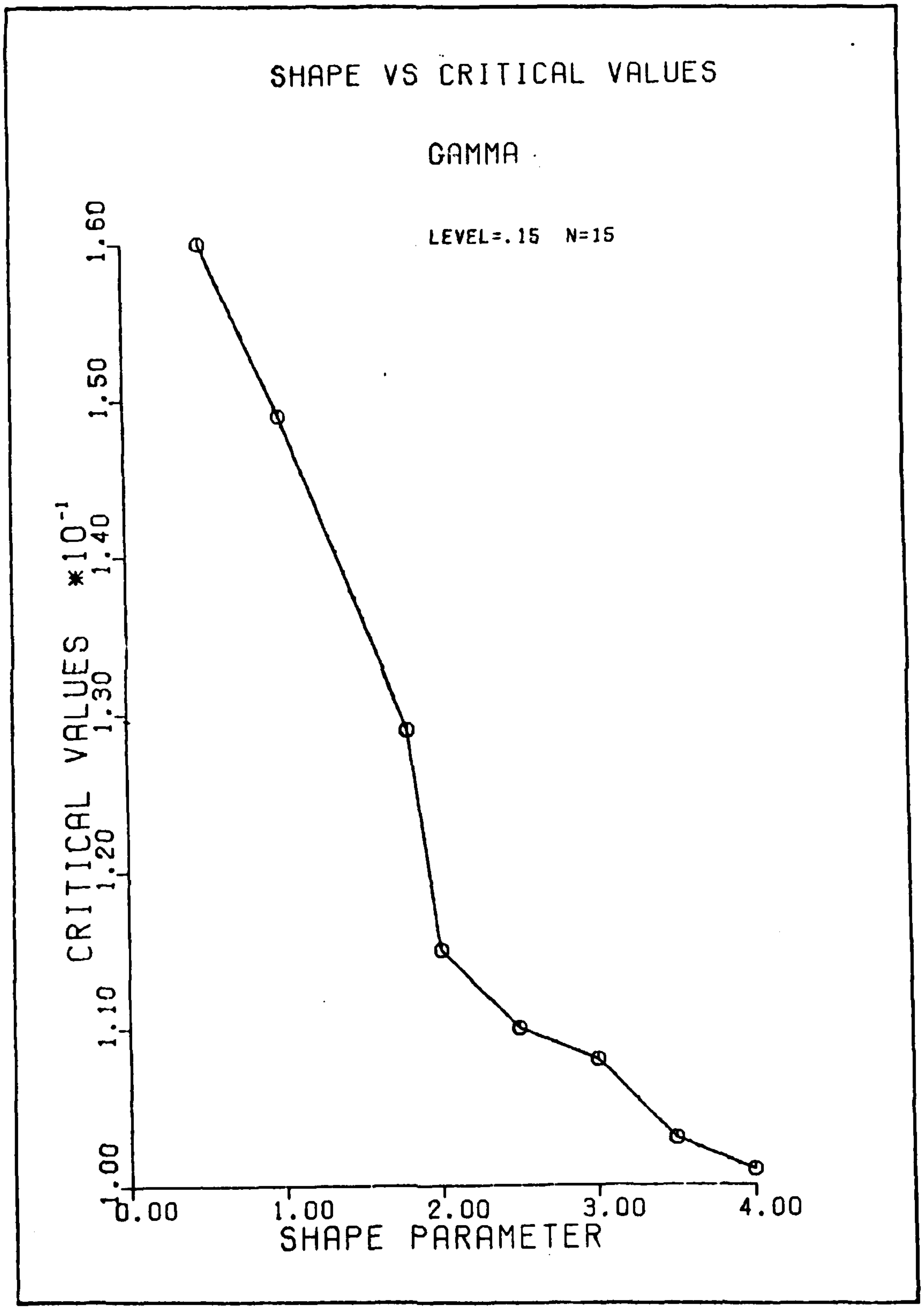


FIG 87. Shape vs W^2 Critical Values, Level = .15, n = 15

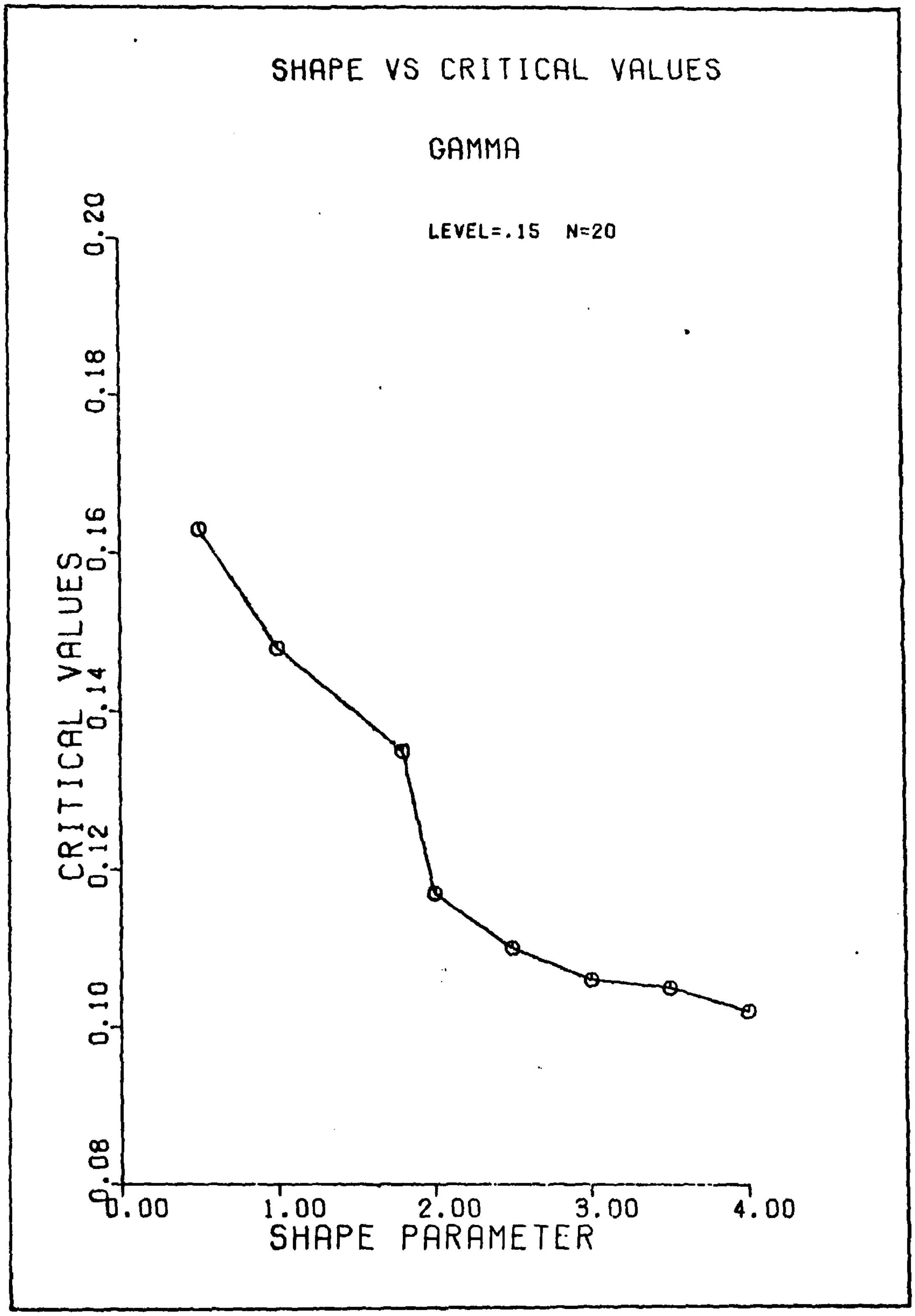


FIG 88. Shape vs S^2 Critical Values, Level = .15, n = 20

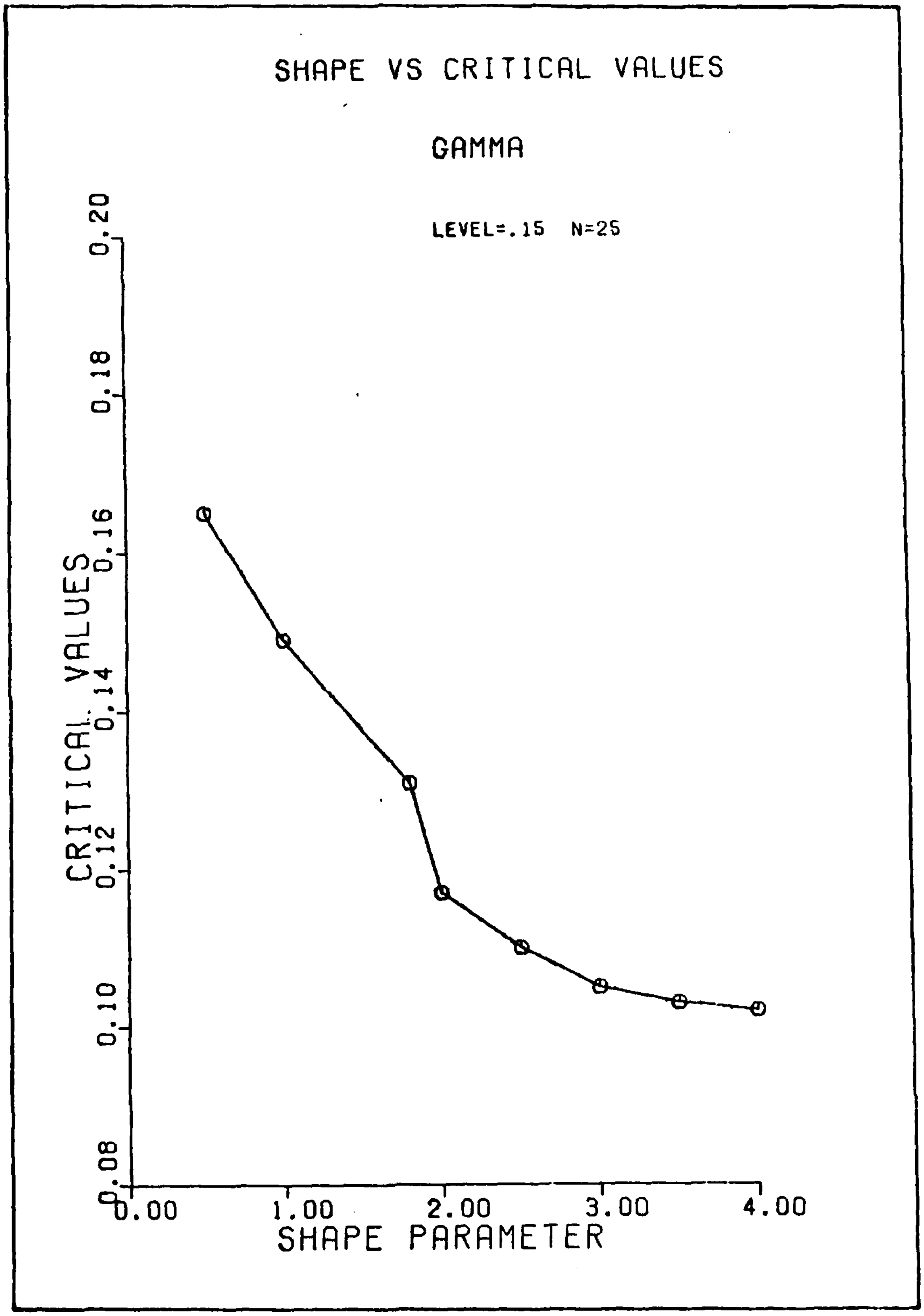


Fig 89. Shape vs A^2 Critical Values, Level = .15, n = 25

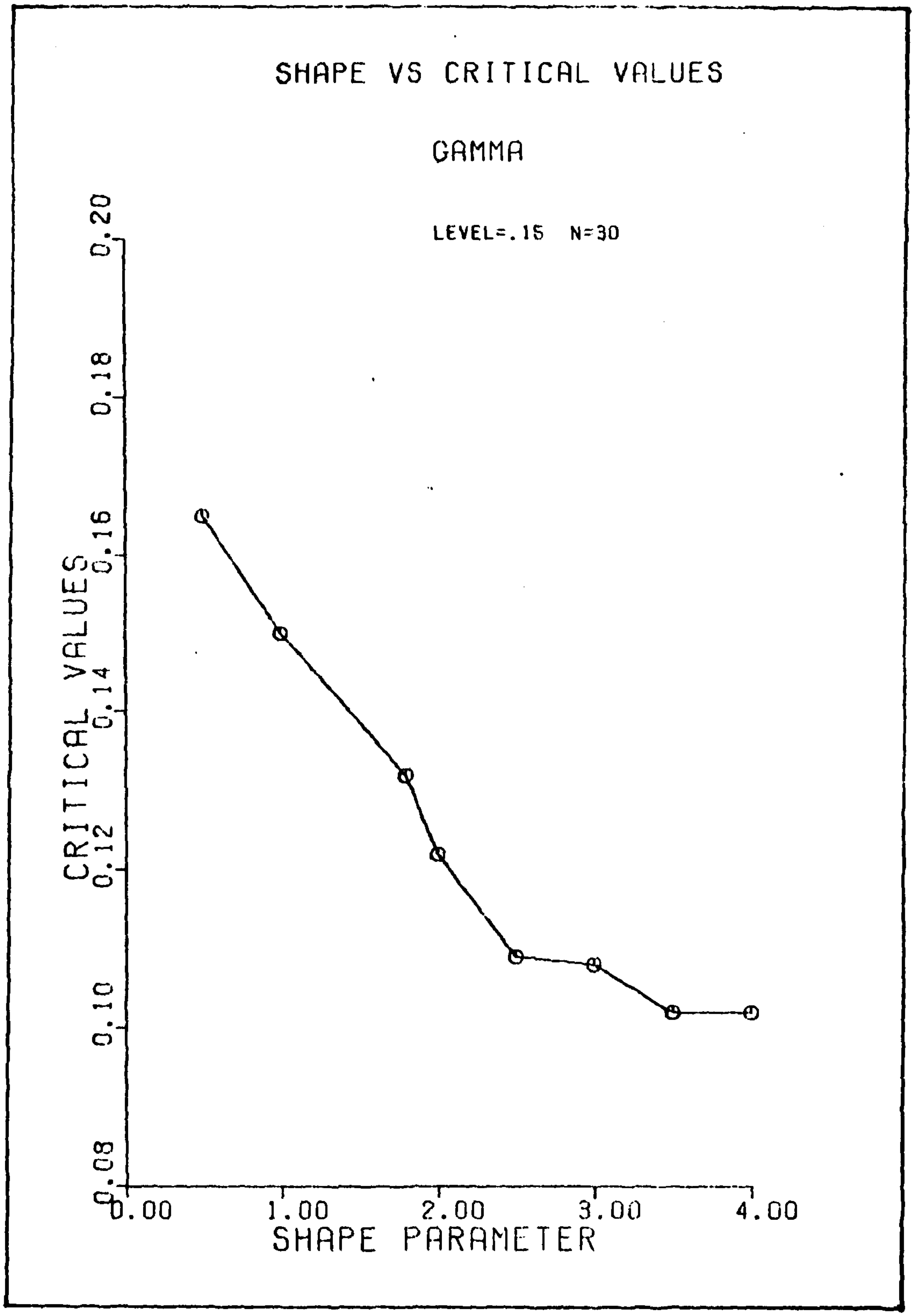


FIG 98. Shape vs W^2 Critical Values, Level = .15, n = 30

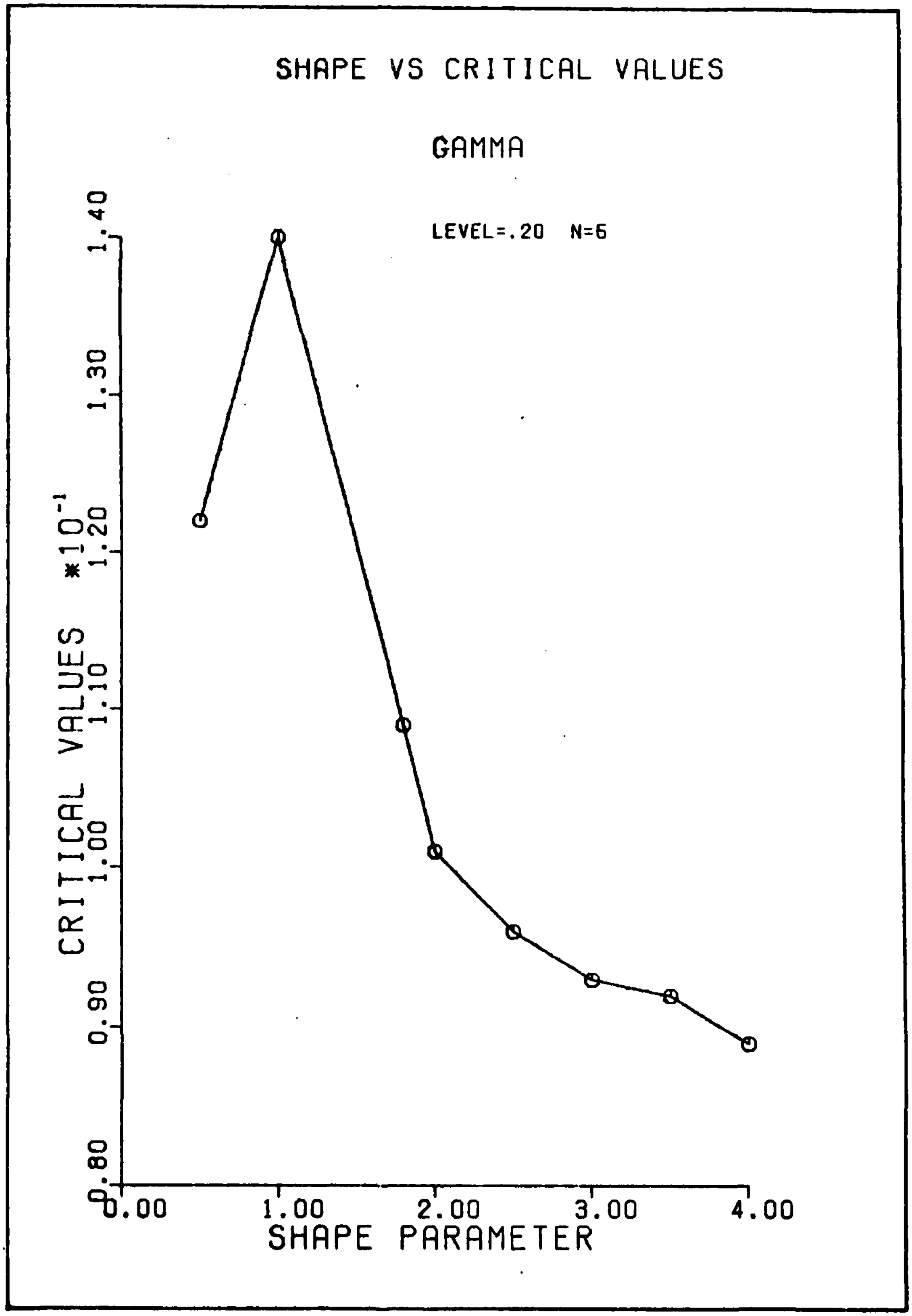


FIG 91. Shape vs W^2 Critical Values, Level = .20, n = 5

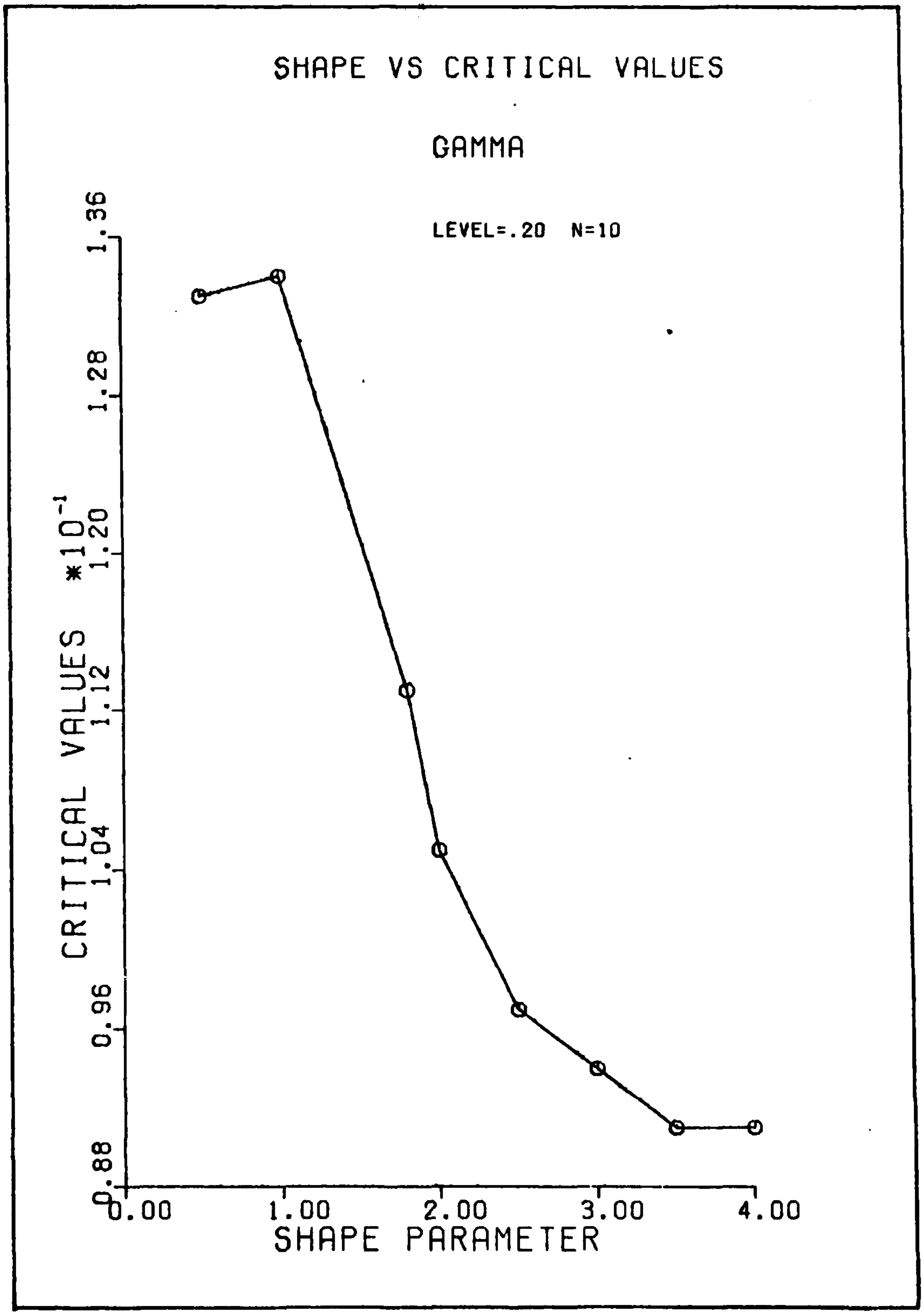


FIG 92. Shape vs W^2 Critical Values, Level = .20, n = 10

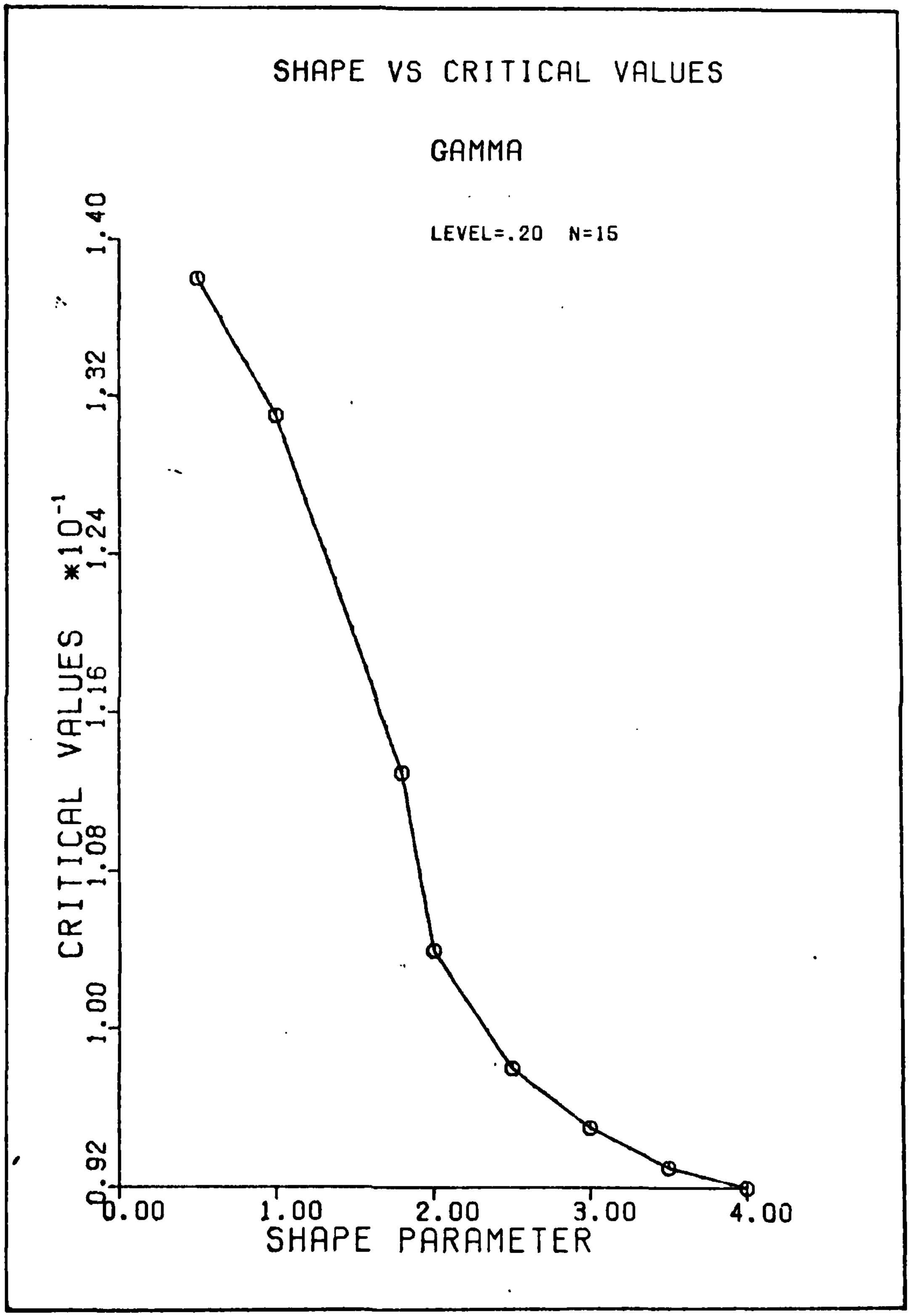


FIG 93. Shape vs W^2 Critical Values, Level = .20, n = 15

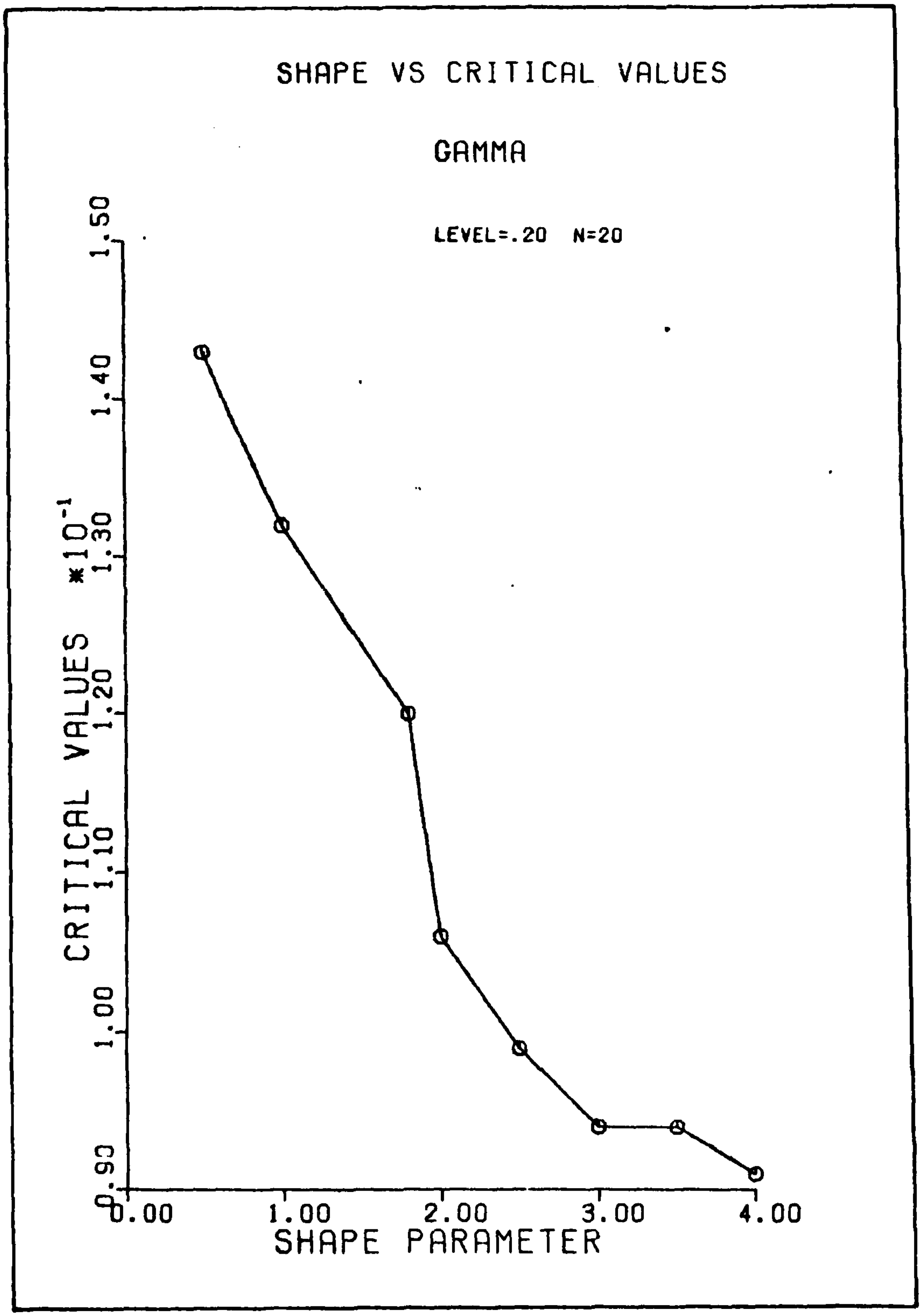


FIG 94. Shape vs W^2 Critical Values, Level = .20, n = 20

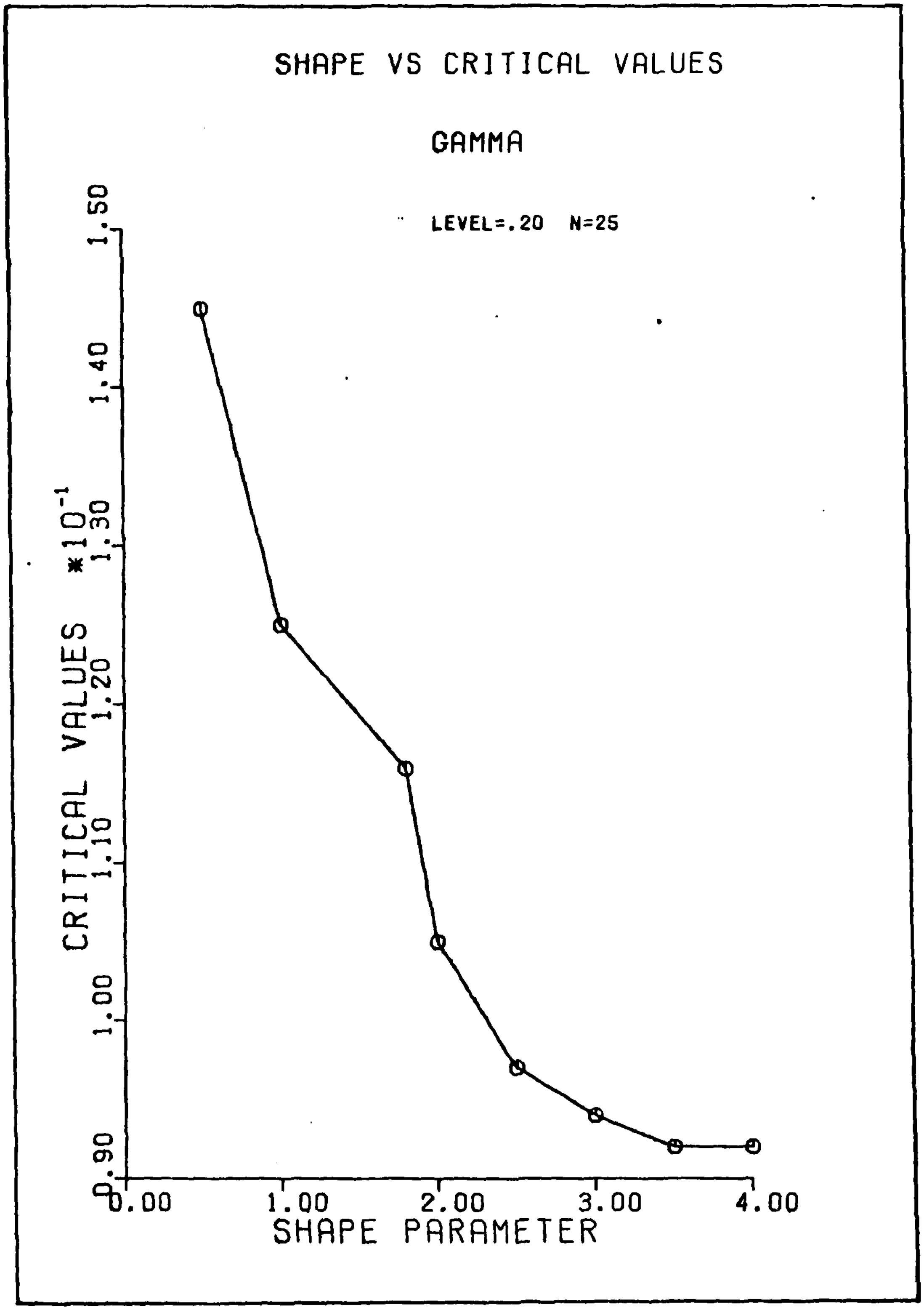


FIG 95. Shape vs W^2 Critical Values, Level = .20, n = 25

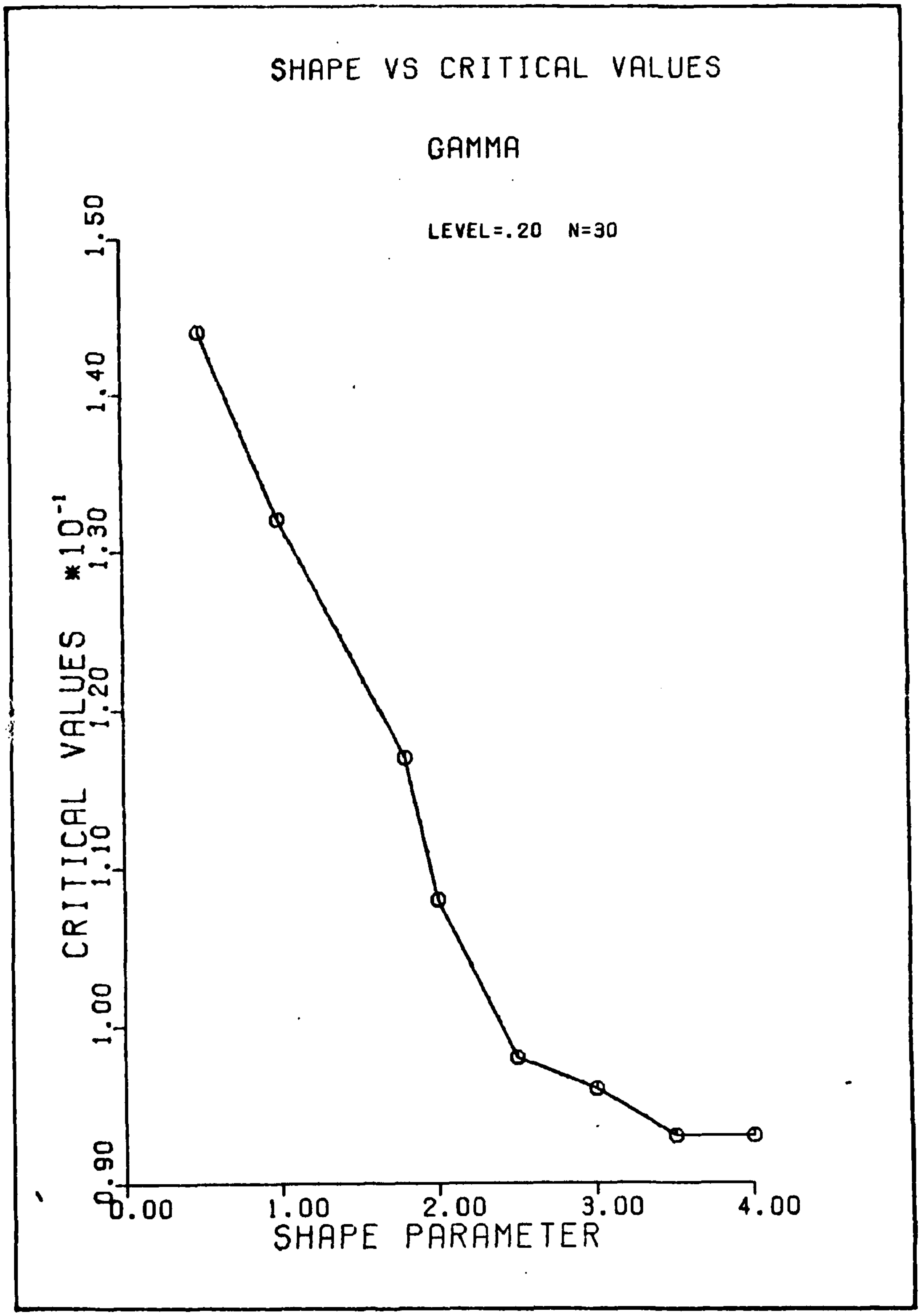


FIG 96. Shape vs W^2 Critical Values, Level = .20, n = 30

APPENDIX G

Computer Programs

```
      PROGRAM AD
C     ****************************************************************
C     ****************************************************************
C     *THIS PROGRAM GENERATES THE MODIFIED A-D STATISTICS            *
C     *5000 REPS                                                      *
C     THE TABLES GENERATED ARE VALID FOR THE GAMMA DISTRIBUTION      *
C     *N = SAMPLE SIZE = 5,(5),30                                     *
C     *SS1 = 0 IF SCALE PARAMETER (THETA) IS KNOWN                    *
C     *SS1 = 1 IF THETA IS TO BE ESTIMATED                            *
C     *SS2 = 0 IF SHAPE PARAMETER (K) IS KNOWN                        *
C     *SS2 = 1 IF K IS TO BE ESTIMATED                                *
C     *SS3 = 0 IF LOCATION (C) IS KNOWN                               *
C     *SS3 = 1 IF C IS TO BE ESTIMATED                                *
C     *C1 = INITIAL ESTIMATE OF C (OR KNOWN VALUE)                    *
C     *T1 = INITIAL ESTIMATE OF THETA (OR KNOWN VALUE)                *
C     *A1 = INITIAL ESTIMATE OF ALPHA (OR KNOWN VALUE)                *
C     ****************************************************************
C     ****************************************************************
      COMMON/VALUE/P(100)
      COMMON/RAY/T(100)
      COMMON/MANA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
      DOUBLE PRECISION DSEED,T,C1,T1,A1
      DIMENSION FX(60),AA(5000),XX(5002),YY(5002),G(50)
      INTEGER REP,PP
      DSEED=15000.0D0
      MR=0
      NONE=0
      NZERO=0
      REP=5002
      NOS=REP-2
      NUM=REP-2
C
C
C     *CALCULATES 5000 PLOTTING POSITIONS ON INTERVAL (0,1) AT (I-.5)/N
C     *THESE ARE USED FOR INTERPOLATIONS ON PERCENTILES
C
      YY(1)=0.
      YY(REP)=1.
      DO 405 L=2,REP-1
        YY(L)=((L-1)-.5)/NOS
405   CONTINUE
C     *END OF PLOTTING POSITIONS ROUTINE
      READ*,SS1,SS2,SS3,C1,T1,A1
      PRINT*,SS1,SS2,SS3,C1,T1,A1
      PRINT*
      PRINT*
      PRINT*
      PRINT'(2X,A,F5.1)','SHAPE=',A1
      PRINT'(2X,A)'     ,'-----------'
      PRINT*
C     *SAMPLE SIZE ASSIGNED HERE
      DO 100 PP=15,15
      N=PP
      M=N
C     *LOOP FOR MONTE CARLO SIMULATION STARTS HERE
      DO 99 KK=1,5000
      CALL GGAMR(DSEED,A1,N,G,P)
```

```
      DO 719 IK=1,N
      P(IK)=1.*P(IK)*10.
719   CONTINUE
      CALL VSRTA(P,N)
      DO 3 II=1,N
      T(II)=P(II)
3     CONTINUE
      CALL GAMMA(CSJ,TSJ,ASJ)
C     *CALCULATES ESTIMATED F(X) FOR EACH SAMPLE POINT
      DO 333 L=1,N
      W=(P(L)-CSJ)/TSJ
      X1=ASJ
      CALL MDGAM(W,X1,PROB,IER)
      FX(L)=PROB
      IF(FX(L).EQ.0.)THEN
      FX(L)=FX(L)+.0001
      NZERO=NZERO+1
      END IF
      IF(FX(L).EQ.1.)THEN
      FX(L)=FX(L)-.0001
      NONE=NONE+1
      END IF
333   CONTINUE
      WAD=0.
      XN=N
      DO 500 I=1,N
      XI=I
      WAD=WAD+(2.*XI-1.)*(LOG(FX(I))+LOG(1.-FX(N+1-I)))
500   CONTINUE
      WAD=(-WAD/XN)-XN
      AA(KK)=WAD
99    CONTINUE
      CALL VSRTA(AA,5000)
      DO 400 L=1,REP-2
      XX(L+1)=AA(L)
400   CONTINUE
      CALL ENDPT(XX,YY,REP,NUM)
C
C     *PRINTS PERCENTILES
C
      PRINT'(2X,A,I2)','FOR N = ',PP
      PRINT'(2X,A)','----------'
      PRINT*
      DO 410 J=80,95,5
      DO 420 II=1,REP
      I=REP+1-II
      IF(YY(I).LT.(J/100.0))THEN
      SLOPE=(YY(I+1)-YY(I))/(XX(I+1)-XX(I))
      ZZ=-SLOPE*XX(I)+YY(I)
      PRINT'(2X,A,I2,A,F9.4)',
     1'THE',J,'TH PERCENTILE IS',
     2((J/100.)-ZZ)/SLOPE
      PRINT*
      GO TO 410
      END IF
420   CONTINUE
410   CONTINUE
```

```
      DO 430 AK=1,REP
      K=REP+1-AK
      IF(YY(K).LT..99)THEN
        GO TO 999
      END IF
430   CONTINUE
999   SLOPE=(YY(K+1)-YY(K))/(XX(K+1)-XX(K))
      ZZ=-SLOPE*XX(K)+YY(K)
      PRINT'(2X,A,F9.4)',
     1'THE 99TH PERCENTILE IS',
     2(.99-ZZ)/SLOPE
      PRINT*
      PRINT*
      PRINT*
      PRINT*
100   CONTINUE
      PRINT'(2X,F9.4,4X,F9.4,4X,F9.4)',CSJ,TSJ,ASJ
      END




C
C
C
C    *SUBROUTINE TO EVALUATE ENDPOINTS
C
C
C
      SUBROUTINE ENDPT(XX,YY,REP,NUM)
      INTEGER REP
      DIMENSION XX(REP),YY(REP)
      SLOPE=(YY(2)-YY(3))/(XX(2)-XX(3))
      B=YY(2)-SLOPE*XX(2)
      V1=-B/SLOPE
      IF(V1.LT.0.)THEN
        V1=0.
      END IF
      XX(1)=V1
      SLOPE=(YY(NUM)-YY(NUM+1))/(XX(NUM)-XX(NUM+1))
      B1=YY(NUM)-SLOPE*XX(NUM)
      V2=(1.-B1)/SLOPE
      XX(REP)=V2
      RETURN
      END
```

```
      PROGRAM P10GG1
C
C
C     THIS PROGRAM GENERATES A POWER STUDY BETWEEN THE FOLLOWING,
C      KS,CRAMER VON-MISES, ANDERSON-DARLING, AND CHI-SQUARE STAT
C     5000 REP
C     THIS POWER STUDY IS VALID FOR THE GAMMA DISTRIBUTION
C     N = SAMPLE SIZE = 25
C     SS1=0 IF SCALE PARAMETER THETA IS KNOWN
C     SS1=1 IF THETA IS TO BE ESTIMATED
C     SS2=0 IF SHAPE (K) IS KNOWN
C     SS2=1 IF (K) IS TO BE ESTIMATED
C     SS3=0 IF LOCATION (C) IS KNOWN
C     SS3=1 IF C IS TO BE ESTIMATED
C     C1=INITIAL ESTIMATE OF C (OR KNOWN VALUE)
C     T1=INITIAL ESTIMATE OF THETA (OR KNOWN VALUE)
C     A1=INITIAL ESTIMATE OF K (OR KNOWN VALUE)
C
C
      COMMON/VALUE/P(100)
      COMMON/RAY/T(100)
      COMMON/MANA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
      DIMENSION FX(60),FIX(60),
     2AAWCVM(5000),AAWAD(5000),AAKS(5000)
      DOUBLE PRECISION DSEED,T,C1,T1,A1
      INTEGER PP
      DSEED=25000.D0
      MR=0
      RWCVM=0.
      RWKS=0.
      RWAD=0.
      NZERO=0
      NONE=0
 710  CONTINUE
      READ ,SS1,SS2,SS3,C1,T1,A1
      PRINT*,SS1,SS2,SS3,C1,T1,A1
      PRINT
      PRINT*
      PRINT
      PRINT*(2X,A,F5.1)*,*SHAPE=*,A1
      PRINT*(2X,A)*    ,*----------------*
      PRINT*
      PP=10
      N=PP
      M=N
      DO 99 KK=1,5000
      CALL GGNLG(DSEED,N,0.,1.,P)
      DO 719 IK=1,N
      P(IK)=1. P(IK)*10.
 719  CONTINUE
      CALL VSRTA(P,N)
      DO 3 IJ=1,N
      T(IJ)=P(IJ)
 3    CONTINUE
      CALL GAMMA(CSJ,TSJ,ASJ)
      DO 88 L=1,N
      W=(T(L)-CSJ)/TSJ
```

```
      XX=ASJ
      CALL MDGAM(W,XX,PROB,IER)
      FX(L)=PROB
      FIX(L)=FX(L)
      IF (FIX(L).EQ.0.)THEN
      FIX(L)=FIX(L)+.0001
      NZERO=NZERO+1
      END IF
      IF(FIX(L) .EQ. 1.) THEN
      FIX(L)=FIX(L)-.0001
      NONE=NONE+1
      END IF
88    CONTINUE
      WCVM=0.
      XN=N
      WAD=0.
      TOP=0.
      BOT=0.
      DO 500 I=1,N
      XI=I
      RL=I
      IF(RL/XN-FIX(I) .GT. TOP) TOP=RL/XN-FIX(I)
      IF(FIX(I)-(RL-1)/XN .GT. BOT) BOT=FIX(I)-(RL-1)/XN
      WCVM=WCVM+(FX(I)-(2. XI-1.)/(2. XN)) 2
      WAD=WAD+(2. XI-1)*(LOG(FIX(I))+LOG(1-FIX(N+1-I)))
500   CONTINUE
      DIF=TOP
      IF(BOT .GT. DIF)DIF=BOT
      WKS=DIF
      AAKS(KK)=DIF
      IF(WKS .GT. .2732)RWKS=RWKS+1.
      WCVM=WCVM+1./(12.*XN)
      AAWCVM(KK)=WCVM
      IF(WCVM .GT. .1391)RWCVM=RWCVM+1.
      WAD=(-WAD/XN)-XN
      AAWAD(KK)=WAD
      IF(WAD .GT..8031) RWAD=RWAD+1
99    CONTINUE
      CALL VSRTA(AAKS,5000)
      CALL VSRTA(AAWCVM,5000)
      CALL VSRTA(AAWAD,5000)
      PRINT*
      PRINT ,*NZERO= *,NZERO
      PRINT*,*NONE= *,NONE
      PRINT
      PRINT*,*SAMPLE SIZE = *,PP
      PRINT
      PRINT
      PRINT*,*TOTAL REJECTION % FOR K-S= *,RWKS/5000
      PRINT*
      PRINT*,*TOTAL REJECTION % FOR WCVM= *,RWCVM/5000
      PRINT
      PRINT*,*TOTAL REJECTION % FOR WAD= *,RWAD/5000
      PRINT
      PRINT
      PRINT*(2X,F9.4,4X,F9.4,4X,F9.4)*,CSJ,TSJ,ASJ
      END
```

```
C
C
C     *SUBROUTINE GAMMA CALCULATES MLE PARAMETERS
C
C
      SUBROUTINE GAMMA(CSJ,TSJ,ASJ)
      COMMON /RAY/T(100)
      COMMON/MANA/N,SS1,SS2,SS3,M,C1,T1,A1,MR
      DOUBLE PRECISION T,C,THETA,ALPHA,DLT,DLA, AL,DLC,CE,TH,EN,EM,ELNM
      DOUBLE PRECISION EMR,EI,D2T,DT,D2A,DA,D2C,DC,ENS,GAM,GMA,GAMI,GMAI
      DOUBLE PRECISION GMAI2,DEXP,DABS,DLOG,SL,DG,DGI,DGI2,SR,S1,DGAM
      DOUBLE PRECISION DGAMI,EL,CSJ,TSJ,ASJ,C1,T1,A1,DSEED
      DIMENSION C(1100),THETA(1100),ALPHA(1100)
      DIMENSION DLT(50),DLA(50),AL(50),DLC(50),CE(50),TH(50)
      JI=20
      JM=20
      C(1)=C1
      THETA(1)=T1
      ALPHA(1)=A1
9     EN=N
      EM=M
86    ELNM=0.D0
      EMR=MR
      MRP=MR+1
87    NM=N-M+1
      DO 88 I=NM,N
      EI=I
88    ELNM=ELNM+DLOG(EI)
      IF(MR)66,89,109
109   DO 110 I=1,MR
      EI=I
110   ELNM=ELNM-DLOG(EI)
89    DO 63 J=1,1100
      IF (J-1) 66,112,111
111   JJ=J-1
      IF (J-JI)6,139,139
139   IF (J/JM-JJ/JM)6,6,117
117   J2=J-2
      J3=J-3
      IF(SS1)119,119,118
118   D2T=THETA(JJ)-2.D0*THETA(J2)+THETA(J3)
      DT=THETA(JJ)-THETA(J2)
      IF (D2T)135,119,135
135   NT=DABS(DT/D2T)
      GO TO 120
119   NT=999999
120   IF (SS2)122,122,121
121   D2A=ALPHA(JJ)-2.D0*ALPHA(J2)+ALPHA(J3)
      DA=ALPHA(JJ)-ALPHA(J2)
      IF (D2A)136,122,136
136   NA=DABS(DA/D2A)
      GO TO 123
122   NA=999999
123   IF (SS3)125,125,124
124   D2C=C(JJ)-2.D0*C(J2)+C(J3)
      DC=C(JJ)-C(J2)
      IF (C(JJ)+5.D-5-T(1))140,125,125
```

```
  140   IF (C(JJ)-5.D-5)125,125,141
  141   IF (D2C)137,125,137
  137   NC=DABS(DC/D2C)
        GO TO 126
  125   NC=999999
  126   NS=2*MINO(NT,NA,NC)
        IF(NS)6,6,142
  142   IF(NS-999999)138,6,6
  138   ENS=NS
        IF (SS1)127,127,128
  127   THETA(J)=THETA(JJ)
        GO TO 129
  128   THETA(J)=THETA(JJ)+(DT+.25D0*(ENS+1.D0)*D2T)*ENS
        THETA(J)=DMAX1(THETA(J),1.D-4)
  129   IF (SS2)130,130,131
  130   ALPHA(J)=ALPHA(JJ)
        GO TO 132
  131   ALPHA(J)=ALPHA(JJ)+(DA+.25D0*(ENS+1.D0)*D2A)*ENS
        ALPHA(J)=DMAX1(ALPHA(J),1.D-4)
  132   IF (SS3)133,133,134
  133   C(J)=C(JJ)
        GO TO 112
  134   C(J)=C(JJ)+(DC+.25D0*(ENS+1.D0)*D2C)*ENS
        C(J)=DMAX1(C(J),0.D0)
        C(J)=DMIN1(C(J),T(1))
        IF ((1.D0-EMR)*C(J)-T(1))112,6,6
    6   THETA(J)=THETA(JJ)
        IF (SS1)13,13,7
    7   S1=0.D0
        DO 8 I=MRP,M
    8   S1=S1+T(I)-C(JJ)
        IF (N-M+MR)66,73,74
   73   THETA(J)=S1/(EM*ALPHA(JJ))
        GO TO 13
   74   GMA=GAM(ALPHA(JJ))
        KS=0
        DO 108 K=1,50
        KK=K-1
        GMAI=GAMI((T(M)-C(JJ))/THETA(J),ALPHA(JJ))
        GMAI2=GAMI((T(MRP)-C(JJ))/THETA(J),ALPHA(JJ))
        DLT(K)=-EM*ALPHA(JJ)/THETA(J)+S1/(THETA(J)**2)+(EN-EM)*(T(M)-C(JJ)
       1)**ALPHA(JJ)*DEXP((C(JJ)-T(M))/THETA(J))/(THETA(J)**(ALPHA(JJ)+1.D
       20)*(GMA-GMAI))+EMR*ALPHA(JJ)/THETA(J)-EMR*(T(MRP)-C(JJ))**ALPHA(JJ
       3)*DEXP((C(JJ)-T(MRP))/THETA(J))/(THETA(J)**(ALPHA(JJ)+1.D0)*GMAI2)
        TH(K)=THETA(J)
        IF (DLT(K))101,13,102
  101   KS=KS-1
        IF (KS+K)105,103,105
  102   KS=KS+1
        IF(KS-K)105,104,105
  103   THETA(J)=.5D0*TH(K)
        GO TO 106
  104   THETA(J)=1.5D0*TH(K)
        GO TO 108
  105   IF (DLT(K)*DLT(KK))107,13,106
  106   KK=KK-1
        GO TO 105
```

```
107   THETA(J)=TH(K)+DLT(K)*(TH(K)-TH(KK))/(DLT(KK)-DLT(K))
      IF (DABS(THETA(J)-TH(K))-1.D-4)13,13,109
108   CONTINUE
13    ALPHA(J)=ALPHA(JJ)
14    IF (SS2)44,44,15
15    SL=0.D0
      DO 16 I=MRP,M
16    SL=SL+DLOG(T(I)-C(JJ))
      KS=0
      DO 43 K=1,50
      KK=K-1
      GMA=GAM(ALPHA(J))
      IF (N-M+MR)66,30,21
21    GMAI=GAMI((T(M)-C(JJ))/THETA(J),ALPHA(J))
      GMAI2=GAMI((T(MRP)-C(JJ))/THETA(J),ALPHA(J))
30    DG=DGAM(ALPHA(J))
76    IF(N-M+MR)66,77,32
77    DLA(K)=-EM*DLOG(THETA(J))+SL-EN*DG/GMA
      GO TO 78
32    DGI=DGAMI((T(M)-C(JJ))/THETA(J),ALPHA(J))
      DGI2=DGAMI((T(MRP)-C(JJ))/THETA(J),ALPHA(J))
38    DLA(K)=-EM*DLOG(THETA(J))+SL-EN*DG/GMA+(EN-EM)*(DG-DGI)/(GMA-GMAI)
     1+EMR*DLOG(THETA(J))+EMR*DGI2/GMAI2
78    AL(K)=ALPHA(J)
      IF (DLA(K))39,44,40
39    KS=KS-1
      IF (KS+K)70,41,70
40    KS=KS+1
      IF (KS-K)70,42,70
41    ALPHA(J)=.5D0*AL(K)
      GO TO 43
42    ALPHA(J)=1.5D0*AL(K)
      GO TO 43
70    IF (DLA(K)*DLA(KK))72,44,71
71    KK=KK-1
      GO TO 70
72    ALPHA(J)=AL(K)+DLA(K)*(AL(K)-AL(KK))/(DLA(KK)-DLA(K))
      IF (DABS(ALPHA(J)-AL(K))-1.D-4)44,44,43
43    CONTINUE
44    C(J)=C(JJ)
85    IF (SS3)112,112,45
45    IF (1.D0-ALPHA(J))79,143,143
143   IF (SS1+SS2)57,57,79
79    IF (N-M)66,83,46
46    GMA=GAM(ALPHA(J))
83    KS=0
      DO 56 K=1,50
      KK=K-1
      SR=0.D0
      DO 69 I=MRP,M
69    SR=SR+1.D0/(T(I)-C(J))
      IF (N-M+MR)66,80,81
80    DLC(K)=(1.D0-ALPHA(J))*SR+EM/THETA(J)
      GO TO 82
81    GMAI=GAMI((T(M)-C(J))/THETA(J),ALPHA(J))
      GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
      DLC(K)=(1.D0-ALPHA(J))*SR+(EM-EMR)/THETA(J)+(EN-EM)*(T(M)-C(J))**(
```

```
     1ALPHA(J)-1.D0)*DEXP(-(T(M)-C(J))/THETA(J))/(THETA(J)**ALPHA(J)*(GM
     2A-GMAI))-EMR*(T(MRP)-C(J))**(ALPHA(J)-1.D0)*DEXP(-(T(MRP)-C(J))/TH
     3ETA(J))/(THETA(J)**ALPHA(J)*GMAI2)
82    CE(K)=C(J)
51    IF (DLC(K))90,112,91
90    KS=KS-1
      IF (KS+K)54,52,54
91    KS=KS+1
      IF (KS-K)54,53,54
52    C(J)=.5D0*CE(K)
      GO TO 68
53    C(J)=CE(K)+.5D0*(T(1)-CE(K))
      GO TO 68
54    IF (DLC(K)*DLC(KK))67,112,55
55    KK=KK-1
      GO TO 54
67    C(J)=CE(K)+DLC(K)*(CE(K)-CE(KK))/(DLC(KK)-DLC(K))
68    IF (DABS(C(J)-CE(K))-1.D-4)112,112,56
56    CONTINUE
      GO TO 112
57    C(J)=T(1)
112   IF (MR)66,113,58
113   DO 115 I=1,M
      IF (C(J)+1.D-4-T(I))116,114,114
114   MR=MR+1
115   C(1)=T(1)
116   IF (MR)66,58,86
58    S1=0.D0
      SL=0.D0
      DO 92 I=MRP,M
      S1=S1+T(I)-C(J)
92    SL=SL+DLOG(T(I)-C(J))
      GMA=GAM(ALPHA(J))
      IF(N-M+MR)66,98,96
96    GMAI=GAMI((T(M)-C(J))/THETA(J),ALPHA(J))
      GMAI2=GAMI((T(MRP)-C(J))/THETA(J),ALPHA(J))
98    EL=ELNM-EM*DLOG(GMA)-EM*ALPHA(J)*DLOG(THETA(J))+(ALPHA(J)-1.D0)*SL
     1-S1/THETA(J)
      IF (N-M+MR)66,100,99
99    EL=EL+(EN-EM)*(DLOG(GMA-GMAI)-DLOG(GMA))
     1+EMR*ALPHA(J)*DLOG(THETA(J))+EMR*DLOG(GMAI2)
100   CSJ=C(J)
      TSJ=THETA(J)
      ASJ=ALPHA(J)
      IF (J-2)63,60,60
60    IF (DABS(C(J)-C(JJ))-1.D-4)51,61,63
61    IF (DABS(THETA(J)-THETA(JJ))-1.D-4)62,62,63
62    IF (DABS(ALPHA(J)-ALPHA(JJ))-1.D-4)4,4,63
63    CONTINUE
4     CONTINUE
66    RETURN
      END
```

FUNCTION GAM 74/74 OPT=0 FTN 5.1+529

```
C     ****************************************************************
C     ****************************************************************
      DOUBLE PRECISION FUNCTION GAM(Y)
      DOUBLE PRECISION G,Z,DLOG,DEXP,Y
      Z=Y
      G=0.D0
1     IF (Z-9.D0)2,2,3
2     G=G-DLOG(Z)
      Z=Z+1.D0
      GO TO 1
3     GAM=G+(Z-.5D0)*DLOG(Z)-Z+.5D0*DLOG(2.D0*3.141592653589793D0)+1.D0/
     1(12.D0*Z)-1.D0/(360.D0*Z**3)+1.D0/(1260.D0*Z**5)-1.D0/(1680.D0*Z**
     27)+1.D0/(1188.D0*Z**9)-691.D0/(360360.D0*Z**11)+1.D0/(156.D0*Z**13
     3)
      GAM=DEXP(GAM)
      RETURN
      END
```

FUNCTION DGAM 74/74 OPT=0 FTN 5.1+529

```
      DOUBLE PRECISION FUNCTION DGAM(Y)
      DOUBLE PRECISION DG,Z,Y,DLOG,GAM
      Z=Y
      DG=0.D0
1     IF (Z-9.D0)2,2,3
2     DG=DG-1.D0/Z
      Z=Z+1.D0
      GO TO 1
3     DGAM=DG+(Z-.5D0)/Z+DLOG(Z)-1.D0-1.D0/(12.D0*Z**2)+1.D0/(120.D0*Z**
     1     4)-1.D0/(252.D0*Z**6)+1.D0/(240.D0*Z**8)-1.D0/(132.D0*Z**10)
     2     +691.D0/(32760.D0*Z**12)-1.D0/(12.D0*Z**14)
      DGAM=DGAM*GAM(Y)
      RETURN
      END
```

FUNCTION DGAMI 74/74 OPT=0 FTN 5.1+529

```
      DOUBLE PRECISION FUNCTION DGAMI(W,Z)
      DOUBLE PRECISION U,V,W,Z,SU,ELL
      DIMENSION U(50),V(50)
      U(1)=W**Z*DLOG(W)/Z
      V(1)=W**Z/Z**2
      SU=U(1)-V(1)
      DO 1 L=2,50
      LL=L-1
      ELL=LL
      U(L)=(-U(LL)*W/ELL)*(Z+ELL-1.D0)/(Z+ELL)
      V(L)=-V(LL)*W*(Z+ELL-1.D0)**2/((Z+ELL)**2*ELL)
1     SU=SU+U(L)-V(L)
      DGAMI=SU
      RETURN
      END
```

```
      DOUBLE PRECISION FUNCTION GAMI(W,Z)
      DOUBLE PRECISION U,W,Z,SU,ELL
      DIMENSION U(50)
      U(1)=W**Z/Z
      SU=U(1)
      DO 1 L=2,50
      LL=L-1
      ELL=LL
      U(L)=(-U(LL)/ELL)*W*(Z+ELL-1.D0)/(Z+ELL)
1     SU=SU+U(L)
      GAMI=SU
      RETURN
      END
```

Vita

Philip John Viviano was born on 15 September 1948 in Saint Louis, Missouri. He graduated from Saint Paul High School in Highland, Illinois in 1966. He received a bachelor's degree in Applied Mathematics from Southern Illinois University in Edwardsville in 1971. He attended Officers Training School and was commissioned as a USAF officer in 1977. He worked as a planning analyst for the Deputy for Development Plans, Electronic Systems Division, Hanscom Air Force Base. He then entered the School of Engineering, Air Force Institute of Technology in June 1981.

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END

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